MCI 13.34h

MARINECORPS INSTITUTE

MATH FOR MARINES

MARINE BARRACKS
WASHINGTON, DC

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IN REPLY REFER TO:
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MCI 1334H MATH FOR MARINES

1. Purpose. MCI 01334H, *Math for Marines*, has been published to provide instruction to all Marines.

2. Scope. This course provides a math review to include algebraic equations, finding area and volume of basic geometric shapes, and use of the Pythagorean theorem. The course presents the history and principles behind each subject.

3. Applicability. This course is intended for instructional purposes only. This course is designed for all Marines.

4. Recommendations. Comments and recommendations on the contents of the course are invited and will aid in subsequent course revisions. Please complete the student suggestion form located at the end of the text and return to

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Table of Contents

Continued on next page

Table of Contents, Continued

Student Information

Number and Title	MCI 1334H MATH FOR MARINES
Study Hours	22
Course Materials	Tex, Protractor
Review Agency	Marine Corps Institute Washington Navy Yard 912 Poor Street SE Washington, DC 20391-5680
Reserve Retirement Credits (RRC)	7 RRCs
ACE	This course is scheduled for review by the American Council on Education during 1992.
Assistance	For administrative assistance, have your training officer or NCO use the Unit Activity Report (UAR) or MCI *Hotline*: DSN 325-XXXX or Commercial (202) 685-XXXX/XXXX. Marines worldwide may call toll free on 1-800-MCI-USMC.

Study Guide

Congratulations

Congratulations on your enrollment in a distance training course from the Distance Learning Technology Department (DLTD) of the Marine Corps Institute (MCI). Since 1920, the Marine Corps Institute has been helping tens of thousands of hard-charging Marines, like you, improve their technical job performance skills through distance training. By enrolling in this course, you have shown a desire to improve the skills you have and master new skills to enhance your job performance. The distance training course you have chosen, MCI 1334H, *Math for Marines,* provides a math review to include algabraic equations, finding area and volume of basic geometric shapes, and use of the Pythagorean theorem. The course presents the history and principles behind each subject.

Your Personal Characteristics

- **YOU ARE PROPERLY MOTIVATED.** You have made a positive decision to get training on your own. Self-motivation is perhaps the most important force in learning or achieving anything. Doing whatever is necessary to learn is motivation. You have it!

- **YOU SEEK TO IMPROVE YOURSELF.** You are enrolled to improve those skills you already possess, and to learn new skills. When you improve yourself, you improve the Corps!

- **YOU HAVE THE INITIATIVE TO ACT.** By acting on your own, you have shown you are a self-starter, willing to reach out for opportunities to learn and grow.

- **YOU ACCEPT CHALLENGES.** You have self-confidence and believe in your ability to acquire knowledge and skills. You have the self-confidence to set goals and the ability to achieve them, enabling you to meet every challenge.

- **YOU ARE ABLE TO SET AND ACCOMPLISH PRACTICAL GOALS.** You are willing to commit time, effort, and the resources necessary to set and accomplish your goals. These professional traits will help you successfully complete this distance training course.

Continued on next page

Beginning Your Course

Before you actually begin this course of study, read the student information page. If you find any course materials missing, notify your training officer or training NCO. If you have all the required materials, you are ready to begin.

To begin your course of study, familiarize yourself with the structure of the course text. One way to do this is to read the table of contents. Notice the table of contents covers specific areas of study and the order in which they are presented. You will find the text divided into several study units. Each study unit is comprised of two or more lessons, lesson exercises, and finally, a study unit exercise.

Leafing Through the Text

Leaf through the text and look at the course. Read a few lesson exercise questions to get an idea of the type of material in the course. If the course has additional study aids, such as a handbook or plotting board, familiarize yourself with them.

The First Study Unit

Turn to the first page of study unit 1. On this page you will find an introduction to the study unit and generally the first study unit lesson. Study unit lessons contain learning objectives, lesson text, and exercises.

Reading the Learning Objectives

Learning objectives describe in concise terms what the successful learner, you, will be able to do as a result of mastering the content of the lesson text. Read the objectives for each lesson and then read the lesson text. As you read the lesson text, make notes on the points you feel are important.

Completing the Exercises

To determine your mastery of the learning objectives and text, complete the exercises developed for you. Exercises are located at the end of each lesson, and at the end of each study unit. Without referring to the text, complete the exercise questions and then check your responses against those provided.

Continued on next page

Continuing to March

Continue on to the next lesson, repeating the above process until you have completed all lessons in the study unit. Follow the same procedures for each study unit in the course.

Seeking Assistance

If you have problems with the text or exercise items that you cannot solve, ask your training officer or training NCO for assistance. If they cannot help you, request assistance from your MCI distance training instructor by completing the course content assistance request form located at the back of the course.

Preparing for the Final Exam

To prepare for your final exam, you must review what you learned in the course. The following suggestions will help make the review interesting and challenging.

- **CHALLENGE YOURSELF.** Try to recall the entire learning sequence without referring to the text. Can you do it? Now look back at the text to see if you have left anything out. This review should be interesting. Undoubtedly, you'll find you were not able to recall everything. But with a little effort, you'll be able to recall a great deal of the information.

- **USE UNUSED MINUTES.** Use your spare moments to review. Read your notes or a part of a study unit, rework exercise items, review again; you can do many of these things during the unused minutes of every day.

- **APPLY WHAT YOU HAVE LEARNED.** It is always best to use the skill or knowledge you've learned as soon as possible. If it isn't possible to actually use the skill or knowledge, at least try to imagine a situation in which you would apply this learning. For example make up and solve your own problems. Or, better still, make up and solve problems that use most of the elements of a study unit.

Continued on next page

Preparing for the Final Exam, continued	• **USE THE "SHAKEDOWN CRUISE" TECHNIQUE.** Ask another Marine to lend a hand by asking you questions about the course. Choose a particular study unit and let your buddy "fire away." This technique can be interesting and challenging for both of you! • **MAKE REVIEWS FUN AND BENEFICIAL.** Reviews are good habits that enhance learning. They don't have to be long and tedious. In act, some learners find short reviews conducted more often prove more beneficial.
Tackling the Final Exam	When you have completed your study of the course material and are confident with the results attained on your study unit exercises, take the sealed envelope marked **"FINAL EXAM"** to your unit training NCO or training officer. Your training NCO or officer will administer the final exam and return the exam the answer sheet to MCI for grading. Before taking your final exam.
Completing Your Course	The sooner you complete your course, the sooner you can better yourself by applying what you've learned! HOWEVER--you do have 2 years from the date of enrollment to complete this course. If you need an extension, please complete the Student Request/Inquiry Form (MCI-R11) located at the back of the course and deliver it to your training officer or training NCO.
Graduating!	As a graduate of this distance training course and as a dedicated Marine, your job performance skills will improve, benefiting you, your unit, and the Marine Corps.

Semper Fidelis!

STUDY UNIT 1

NUMBER SYSTEMS AND OPERATIONS

Introduction. Every Marine, from a scout in a fireteam to the Commandant, uses numbers in some form every day. Numbers are an inescapable item in our way of life. Some common uses of numbers may be any one of the following:

All Marines will carry 3 days' rations and ammunition.

How much fuel will the tank platoon need to travel 127 miles?

Calculate the TNT needed by the engineer platoon to breach the log obstacle.

Convert 123° magnetic azimuth to grid azimuth.

If a 20 mile march took 5 hours to complete, how fast did the company travel?

These are just a few of the everyday situations involving numbers that Marines deal with. Marines need to know how to work with numbers. This study unit will provide you with the ability to identify components of the real number system and basic properties of numbers. You will also be able to identify symbols of grouping and simplify mathematical expressions. Let's first take a look at the history of real numbers.

Lesson 1. HISTORY OF REAL NUMBERS

<u>LEARNING OBJECTIVES</u>

1. Given a list of events, select the single factor that made the Hindu-Arabic number system superior to all others.

2. Given selected purposes, identify the two purposes that numerals or digits have in any number.

1101. Systems of Notation

In order to have an understanding of the systems of notation, you must first look at the principle of one to one correspondence and model groups. Let's see how Charlie Caveman handled these. In ancient times, Charlie had to design a way to keep track of his herd.

The method that he used is basic to our present day mathematics. He simply matched the number of animals with items in a collection of objects. For example, as he let the animals out in the morning he tied a knot in a rope for each animal; when they came back in the evening he matched each knot with an animal. If he had knots left over, he had animals missing. He did the same thing by piling up stones or sticks, each one representing an animal.

As Charlie became more rational, he tired of those processes for accounting for his possessions. His next step toward the beginning of numbers was the formation of model groups. This was an extension of his one-to-one correspondence or matching process. It involved using well-known items from his environment to symbolize a given collection of objects. Each symbol had a characteristic that was typical of the model group. These were exemplified by: the wings of a bird symbolizing two; the leaves of a clover, three; the legs of a sheep, four; the fingers on a hand, five; the petals on a particular flower, six; and so on. In his mind, Charlie replaced that pile of five stones with the number of fingers on his hand. Naturally, the evolution brought about using the name for the model group instead of the actual symbol. Although it still was not counting, the model group was a major step toward it. It simply provided a recognition of the cardinal (how many) value of a number. Later, when man started to systematize his model groups and organize them in sequence (each succeeding the other by one), he was on his way to counting. Let's look at the different systems of notation and see how this was accomplished.

a. Egyptian number system. Anthropologists have shown us that all of the early civilized cultures had number systems and counting. The Egyptians used a system that started with tally marks which were continued until 10 which was replaced by the symbol for a heelbone. Each succeeding power of 10 was replaced by a different symbol. The system was based on addition and repetition. That is, add up the repeated symbols (fig 1-1).

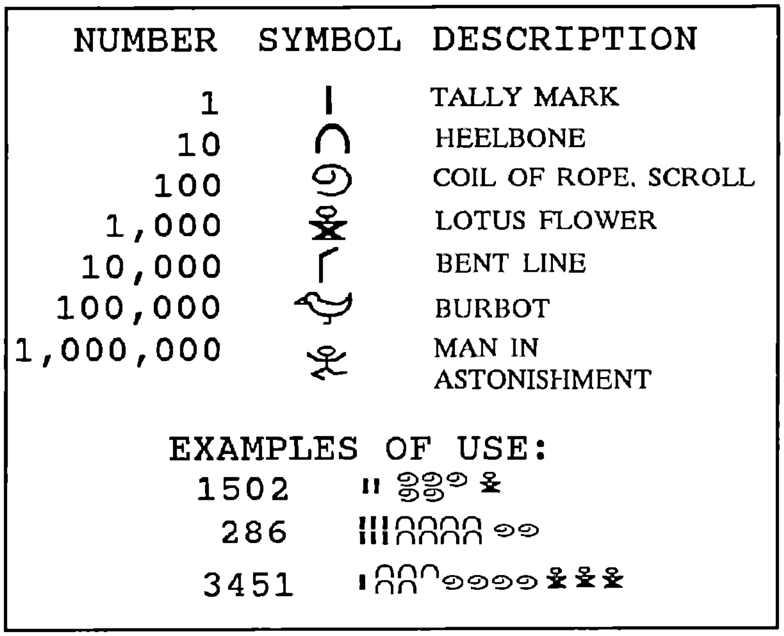


Fig 1-1. Symbols for Egyptian Numbers.

b. <u>Roman number system</u>. Most of us are familiar with the Roman system of notation because it is still taught in our schools and is used to some extent for decoration on monuments or buildings, and for dates. Basically, this system is also founded on the principles of addition and repetition. The Roman system greatly reduced the symbolism that was required in the Egyptian system, but it still did not approximate the value of the Hindu-Arabic number system.

c. <u>Hindu-Arabic number system</u>. The Hindu-Arabic system came into being possibly as early as 500 or 600 BC. The nomadic Arabian tribes picked up the Hindu notation in their travels and carried it to Europe. It was several hundred years, however, before it replaced the Roman system. This system differs from the Egyptian and Roman systems in that a different symbol is used for each number. It is similar to the other systems because it is based on addition, meaning that any number's value is a sum of its parts. It is similar to the Egyptian system because there is a compounding at each power of ten. This simply means that the Hindu-Arabic system is a decimal system. An important similarity to the Roman system is the position of the numerals. Moving the symbols within the number changes the value of the number. The unique principle of the Hindu-Arabic system was the idea of place value (how many of what) which no system had previously employed. In its early use, the Hindu-Arabic system had no distinct advantage over the number system of any other culture.

The single factor that made the Hindu-Arabic number system superior to all others was the invention of zero sometime in the 9th century AD. The zero serves as a place holder and a signifier of "no value," or "nothing." When used in this context, the zero becomes significant. The Hindu-Arabic system uses 10 basic symbols: 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. In any number, each of the numerals or digits signifying it has two purposes; one is to show the cardinal value (how many) and the other is to show the place value (that is, how many units, tens of units, hundreds of units, thousands of units, and so on). To show the cardinal value, the symbol 0, conveys the concept of absence or emptiness, the symbol 1, the concept of oneness or singleness and so on. To show the place value, with the aid of the principle of addition, we show the next number after nine as 10 which symbolizes one ten and no units. Here the zero is functioning as a placeholder occupying space in the units column. It shows that there are no units involved in this number. The next number is 11 which is one ten and one unit. The next number is 12 which is one ten and two units. This continues until 99 which is nine tens and nine units after which we must use three digits. The numerals 100 signify one hundred, no tens, and no units. The numerals 463 signify four hundreds, six tens, and three units. The numerals 999 signify nine hundreds, nine tens, and nine units. After this, four digits are needed. As you can see from this, the system is unlimited in size.

Lesson Summary. This lesson provided you with a brief history of real numbers, the single event that made the Hindu-Arabic number system superior to all others, and the two purposes found in any number. Now, let's move forward to Lesson 2 and take a look at the natural numbers.

Lesson 2. NUMBERS AND CLOSURE

LEARNING OBJECTIVES

1. Given selected numbers, identify the natural numbers in the counting process.

2. Given selected mathematical equations, select the example that illustrates the principle of closure.

1201. Natural Numbers

Remember that when Charlie Caveman put the model groups into an organized sequence, this was the forerunner to counting. The successive model groups in his counting system each increased by one the size of the group that preceded it. This was the key to the counting process (increase by one). Now man had the means to count, and the numbers he used to do this counting are called the natural numbers. They are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. . . (the three dots in mathematics indicate "and so on"). Note that the counting or natural numbers do not include zero (fig 1-2).

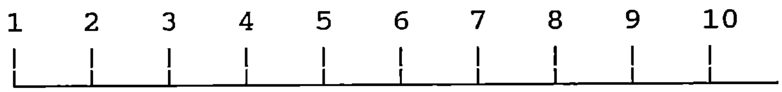


Fig 1-2. Graphical representation of the natural numbers.

In order to understand the difference between the various systems and why they developed, it is necessary to talk about one important principle of numbers called closure.

1202. Principle of Closure

The principle of closure means that, regardless of any operation (addition, subtraction, multiplication, or division) you perform on a number in the rational number system, the result will always be a rational number. Other number principles will be discussed in a later study unit, but closure must be discussed here in order to progress to the other systems. If you add, multiply, subtract, or divide natural numbers, will the sum, product, difference, or quotient be a natural number? Let's take a look at each of these areas and see how closure affects them.

a. Addition. If you add 6 + 8, 10 + 27, 99 + 236, 9872 + 10463, or any two natural numbers, the result will always be a natural number. When this occurs, the system is said to be closed under addition.

b. Multiplication. If you multiply 5 x 6, 33 x 44, 356 x 687, 3422 x 22345, or any two natural numbers, the result will always be a natural number. This is because multiplication is a form of repeated additions, which makes the system closed under multiplication. That is, the product of any two natural numbers will always be a natural number.

c. Subtraction. How about subtraction? Is the difference between any two natural numbers a natural number? Again look over the system and try a few examples. It should not take you long to see that problems such as 7 - 10 and 20 - 25 cannot be answered with a natural number. Thus, natural numbers are not closed under subtraction.

d. Division. An example of 4 ÷ 3 = 1.33..., is enough to show that the natural numbers are not closed under division.

What does all of this mean to you? What is the importance of closure? The early cultures had extremely little use for numbers. Counting, mainly to keep track of possessions, was their primary use. Charlie Caveman had no need for anything other than the natural numbers, but with civilized cultures came the development of towns and the use of money which places a premium on knowledge of numbers. This is one of several reasons why closure is important and that there is a need for other number systems.

Lesson Summary. This lesson provided you with the ability to identify the natural numbers in the counting process and introduced the principle of closure. You learned that natural numbers are not closed under subtraction and division. Closure shows you that the natural numbers are not sufficient to handle all operations. There is a need for other number systems. They will be covered in our next lesson.

Lesson 3. INTEGERS AND RATIONAL NUMBERS

LEARNING OBJECTIVES

1. Given various mathematical problems, select the equation that illustrates integers as a system of positive and negative whole numbers.

2. Given a list of selected mathematical terms, select the one that best illustrates the concept of rational numbers.

1301. System of Whole Numbers

a. Integers. The integers are the system of whole numbers. Of course the natural numbers are whole numbers, but not the complete set that man was to need. With the use of subtraction there came a need for an expanded system. If you recall, it was mentioned that problems such as 7 - 10 and 20 - 25 could not be answered with the natural numbers. The answers to both of these problems require a system that includes positive and negative numbers (fig 1-3). Note that the integers start at and include zero, and extend indefinitely in both directions from that point.

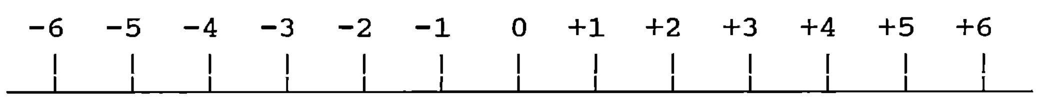


Fig 1-3. Graphical representation of the integers.

If you perform several checks on this system, you will see that it is closed under addition, multiplication, and subtraction; but again, a simple problem such as 3 ÷ 2 = 1.5 is enough to show that the integers are not closed under division.

b. Rational Numbers. Rational numbers are simply numbers that can be expressed as fractions, that is, as the quotient of two integers. Man must have had a need for the rational numbers before the integers, but the integers were discussed first because of their association with subtraction which man probably used very early in his number career. The rational numbers include the integers (5, 6, 7), terminating and repeating decimal fractions (.25 = 1/4, .333 = 1/3), and all fractional numbers.

It should be evident that the system is closed under all four of the basic operations: addition, subtraction, multiplication, and division. That is, any of these operations performed on a number in the rational number system will give us a rational number as the answer.

Natural numbers, the two number systems that we have just mentioned (integers, rational), and one additional system (irrational numbers) make up the real number system (fig 1-4).

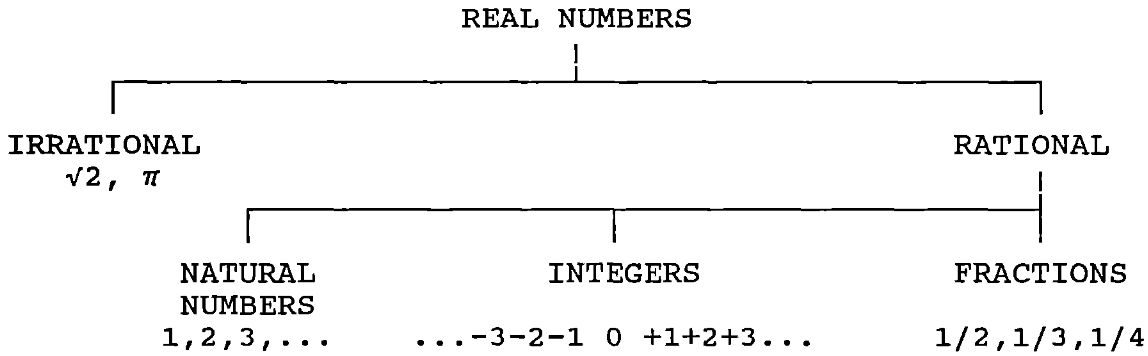


Fig 1-4. The real number system.

The graphical representation of the real numbers looks the same as the one for rational numbers. For our use, the numbers are the same. The irrational numbers are numbers such as $\sqrt{2}$, π, and non-terminating, non-repeating decimals. We will discuss them in a later study unit.

<u>Lesson Summary</u>. This lesson provided you with the information to identify that integers are a system of whole numbers composed of positive and negative numbers, and rational numbers are numbers that can be expressed as fractions.

Lesson 4. PROPERTIES OF NUMBERS

<u>LEARNING OBJECTIVES</u>

Given simple equations, and according to established laws of mathematics:

1. Identify the three axioms of equality.

2. Identify the commutative property with respect to addition and multiplication.

3. Identify the associative property with respect to addition and multiplication.

4. Identify the distributive property with respect to multiplication.

In the pre-1950 era of mathematical instruction, little was ever taught about why you can do some of the things that you do with numbers. Now we are teaching the "why" in addition to the "how." Upon analysis, you will see that much of this "why" is self-evident. This is actually what an "axiom" is -- "a self-evident truth." Since you have been taught how to perform mathematical operations and you now do them without second thought, the introduction of the properties of numbers may seem at first irrelevant and unnecessary. But, this is judgment based on hindsight. Place yourself in the position of the beginner in the study of arithmetic and imagine the value that could be gained by knowing the fundamental properties of a number. Even at this stage of your mathematical education, an understanding of the properties can lead to an increased ability to handle more complicated problems. What you need to understand next are the three axioms of equality. (An equality is simply a statement that two things are equal: a = b.) The axioms of equality are important in the study of equations, and they also have important applications in your work in arithmetic. Let's take a look at them.

1401. Three Axioms of Equality

a. The reflexive property of equality. This is sometimes referred to as the law of identity. It is simply a long-winded way of saying that any number reflects itself or is equal to itself. Stated symbolically, for any number a, a = a.

b. The symmetric property of equality. This states that an equality is reversible. Stated symbolically, for any numbers a and b, if a = b then b = a.

c. The transitive property of equality. This makes it possible to identify two numbers as being equal to a third quantity. Stated symbolically, for any numbers a, b, and c, if a = b and b = c, then a = c.

Now that you understand the axioms of equality, let's take a look at the commutative and associative properties.

1402. Commutative and Associative Laws or Properties

There are certain laws that are obeyed by various number systems. In fact, number systems can be categorized by their conformity to these laws. Two of the basic laws are the commutative and the associative. Let's look at how these laws are applied.

a. The commutative law or property with respect to addition. This law states that the order of addends (numbers to be added) is immaterial; the sum will be unaffected. It doesn't matter whether we add 7 and 3 by writing 7 + 3 or 3 + 7. Stated symbolically, for any numbers a and b, a + b = b + a.

b. The associative law or property with respect to addition. This law states that the order in which addends are grouped or associated is immaterial; again the sum will be unaffected. This pertains to a problem with at least three addends. Addition is a binary operation meaning that only two numbers can be added at one time; however, their sum may be added to another and so on. In the problem 4 + 5 + 8, it doesn't matter whether you add 4 and 5 and then add this sum to 8, whether you add 4 and 8 and this sum to 5, or whether you add 5 and 8 and this sum to 4. The answer will always be 17. Stated symbolically, for any numbers a, b and c, $(a + b) + c = a + (b + c) = (a + c) + b$.

c. The commutative law or property with respect to multiplication. This law is similar to the law for addition. It states that the order of factors (numbers to be multiplied) does not affect the product. In the problem 8 x 7, it doesn't matter whether you multiply 8 x 7 or 7 x 8, the result is 56. Stated symbolically, for any numbers a and b, $a \times b = b \times a$.

d. The associative law or property with respect to multiplication. This law is also similar to the law for addition. It states that in any multiplication problem, the product is unaffected by the combinations in which the factors are grouped. Again, as with addition, multiplication is binary and the groupings will be in pairs. In the problems 4 x 9 x 3, the same product will be found by (4 x 9) x 3, or by 4 x (9 x 3). Stated symbolically, for any numbers a, b and c, $(a \times b) \times c = a \times (b \times c) = (a \times c) \times b$.

1403. Distributive Property for Multiplication

This law or property states that multiplication can be distributed over addition. It can be best explained by use of an example: Multiply 3 times the sum of 4 and 6. One method of doing the problem is to add the 4 and the 6 then multiply this by 3 to get the answer 30: 3 x (4 + 6) = 3 x 10 = 30. Now by the distributive method, you distribute the 3 to each of the addends: 3 x (4 + 6) = (3 x 4) + (3 x 6) = 12 + 18 = 30

Although it may not seem like the best method for this particular problem, it is one of the most used properties in algebra. Remember that this property is for multiplication over addition. The converse is not true; addition is not distributive over multiplication. The examples below show the difference between the two:

$$3 + (4 \times 6) = 3 + 24 = 27$$

$$3 + (4 \times 6) = (3 + 4) \times (3 + 6) = 7 \times 9 = 63$$

Note: (27 does not equal 63)

Stated symbolically the distributive law is, for any numbers a, b, and c, $a \times (b + c) = a \times b + a \times c$.

Lesson Summary. In this lesson you identified the three axioms of equality: reflexive, symmetric, and transitive. You also studied the commutative property for addition and multiplication, the associative property for addition and multiplication, and the distributive property for multiplication. There are many other laws or properties of numbers and all of them are important for various applications; you will cover two more in the next lesson.

Lesson 5. OTHER PROPERTIES OF NUMBERS

LEARNING OBJECTIVE

Given simple equations, solve the equation using the zero and unity elements.

1501. Zero and Unity Elements

The zero and unity elements are the two additional laws and properties of numbers that you will cover; let's take a look at them.

a. Zero element. As pointed out earlier, the invention of zero by the Hindus is what made the Hindu-Arabic number system. Its value as a place holder and a signifier of a nothing value is what set the system apart from others. The zero has several unique properties that come into play in operations with numbers. The first one is in addition.

(1) Addition. Zero is referred to as the additive identity or the identity element for addition. This simply means that when zero is added to a number it preserves the identity of the number: 5 + 0 = 5, 0 + 8 = 8, 10 + 0 = 10, etc. Although fundamental, this property is not always obvious. Let's see how it works with both multiplication and division.

(2) Multiplication. Multiplication with the zero is quite another story. Where adding zero to a number preserves its identify, multiplication of a number by zero destroys its identity, for any number times zero is zero. Stated symbolically, for any number a, a x 0 = 0 x a = 0.

(3) Division. There are two aspects of division with zero. Dividing zero by a number always results in zero. Dividing a number by zero is impossible. These statements can be proved by several examples.

In dividing zero by a number, 0 ÷ 8 = ?, whatever number replaces the question mark must be a number that, when multiplied by 8, will give zero. Remember that in order to check a division problem, multiply the quotient (answer) times the divisor. This will give the dividend. In the above problem, the only possible replacement for the question mark is zero since it is the only number that multiplied by 8 will give zero.

You should see that this will be the same for any number divided into zero. Stated symbolically, for any number a, $0 \div a = 0$.

In dividing zero into a number, $6 \div 0 = ?$, you should see that there is no replacement for the question mark that when multiplied by zero will give 6. This would be the same for any number. This operation is said to be impossible or undefined.

Stated symbolically, for any number a, $a \div 0 =$ impossible. You have just seen the effects of the zero element on numbers; let's now take a look at the effects of the unity element on numbers.

b. Unity element. The number one has many significant properties. One aspect that you want to examine is its identity quality. Where zero is the identity element in addition, one is the identity element in multiplication. That is, any number multiplied by one will be equal to that number, or one will preserve the number's identity. This property is sometimes referred to by the important-sounding title, multiplicative identity. Stated symbolically, for any number a, $1 \times a = a \times 1 = a$.

Division with the number one also has some important aspects. Dividing a number by one preserves its identity in the same manner as multiplying by one. That is, any number divided by one gives us that number. Stated symbolically, for any number a, $a \div 1 = a$.

Dividing one by a number is quite a different story. One divided by a number gives the reciprocal of the number. This is a very important term that you will find throughout your mathematical education. You will have occasion to use it several times in this course. For example, the reciprocal of 15 is 1/15, the reciprocal of 3/4, is 4/3. Note: An important fact is that the product of a number and its reciprocal is one; for example: $4 \times 1/4 = 1$, $15 \times 1/15 = 1$, $3/4 \times 4/3 = 1$, etc.

Lesson Summary. This lesson taught you two more important laws, the zero and unity elements and how they effect the identity of our number system.

Lesson 6. SYMBOLS OF GROUPING

LEARNING OBJECTIVE

Given simple equations using symbols of grouping, simplify the mathematical statements.

1601. Mathematical Punctuation Marks

The symbols of grouping are the punctuation marks of mathematics. They change the meaning of a group of numbers the same way that commas can change the meaning of a group of words. For example, punctuate this sentence:

Pvt. Buttplate said the Gunny is a meathead.

If you are a Gunnery Sergeant you probably did it something like this: "Pvt. Buttplate," said the Gunny, "is a meathead."

On the other hand, you may have completely changed the meaning by punctuating the sentence like this: Pvt. Buttplate said, "The Gunny is a meathead."

You should see from these examples that the placement of punctuation marks is important. Placing symbols of grouping in mathematical equations is equally important. The following examples show the four <u>different symbols of grouping</u> (devices) used:

parenthesis (6-2)

brace {6 - 2}

bracket [6 - 2]

bar or vinculum $\overline{6 - 2}$

Let's look at a problem and see how the use of these devices can clear it up. What is the answer to this problem: 3 x 7 + 4 = ? Is it 25 or 33? There is a proper order of operations for this type of problem, and you will look into this in the next lesson; however, the problem is clarified when you insert a grouping symbol such as a parenthesis: 3 x (7 + 4) = 33. Moving the parenthesis to another position will produce a different result: (3 x 7) + 4 = 25. In the first example, 3 x (7 + 4), you have a multiplication sign included. When a number directly precedes or follows a grouping symbol, multiplication is indicated. For example, 3(6), (3)(6), and (3)6 all indicate to multiply 3 times 6.

Grouping symbols are all used in the same manner. Our original problem could be written in four ways:

$$3\ (7 + 4) = 3 \times 11 = 33$$

$$3\ [7 + 4] = 3 \times 11 = 33$$

$$3\ \{7 + 4\} = 3 \times 11 = 33$$

$$3\ \overline{7 + 4} = 3 \times 11 = 33$$

As you can see, symbols of grouping help simplify mathematical statements, but what happens when you have more than one set of symbols? Our next lesson, Order of Operations, will help answer this question.

Lesson Summary. This lesson taught the punctuation marks for mathematics, better known as the symbols of grouping. They are used to simplify mathematical statements.

Lesson 7. ORDER OF OPERATIONS

LEARNING OBJECTIVE

Given the rules for the order of operations, solve simple equations.

1701. Solving Mathematical Expressions

In solving a mathematical expression it is possible to have more than one answer, but there is only one correct answer. It depends on the order in which you perform the indicated operations of the problem (3 x 7 + 4 = 25 or 33?). In the preceding lesson, you saw how the use of grouping symbols gives a clear indication of the intent of the problem, making it easier to work the problem. Whether or not there are any grouping symbols in a problem, there are rules which dictate the proper sequence of operation. These rules must be followed if you are to arrive at the correct answer:

a. When grouping symbols are used, simplify the parts of the expression first, following the order of operation in rules (b) and (c) listed below.

b. Perform all multiplication or division operations indicated in the expression in the order you come to them, working from left to right.

c. Perform all addition or subtraction operations indicated in the expression in the order you come to them, working from left to right.

Remember, when you have a problem or equation which contains more than one type of grouping symbol, "work from the inside out." This simply means complete <u>all</u> of the indicated operations <u>within</u> the innermost set of symbols <u>first</u>, the next innermost set of symbols second, and so on. When all of the indicated operations within a grouping symbol have been completed, the symbol may be removed from the problem. This will keep the problem from becoming cluttered with unnecessary symbols which could cause you to make an error. Also, neatness and an orderly step-by-step procedure for simplifying a problem will prevent many errors.

Remember, you must do <u>all</u> multiplication and division before any addition and subtraction, and the <u>rules</u> must be applied to each step needed to simplify the problem or equation. Now that you have rules to govern the order of operations, the answer to the expression presented previously is quite easy: 3 x 7 + 4 = ? = 21 + 4 = 25.

Let's look at a few other examples:

$$6 \times 8 - 7 \times 2 = (6 \times 8) - (7 \times 2) = 48 - 14 = 34$$

$$30 \div 10 \times 3 = (30 \div 10)3 = 3 \times 3 = 9$$

These examples show you how important the rules for the <u>order of operations</u> are in solving equations. Remember, when grouping symbols are not given, these rules must be followed if you are to arrive at the correct answer.

<u>Lesson Summary</u>. This lesson demonstrated how to use the rules for order of operations to solve given equations. Remember the rules: Simplify by removing the grouping symbols, multiply or divide, then add or subtract.

- -

Unit Exercise: Complete items 1 through 38 by performing the action required. Check your responses against those listed at the end of this study unit.

1. What single factor made the Hindu-Arabic number system superior?

   a. Ability to perform addition
   b. Compounding of powers of ten
   c. Ability to repeat numbers
   d. Invention of zero

2. The two purposes of the digits in any number is to show the

   a. cardinal and place values.
   b. cardinal and face values.
   c. natural and place values.
   d. natural and face values.

3. Which set consists of entirely natural numbers?

   a. 3, 17, 19.5
   b. 1/2, 2, 10
   c. 0, 1, 2
   d. 3, 15, 27

4. Which example illustrates the principle of closure?

   a. 4 - 9 =
   b. 4 + 9 =
   c. 4 ÷ 9 =
   d. 9 ÷ 4 =

5. Which answer to the example below is an integer?

   a. 3 ÷ 2
   b. 12.6 - 4
   c. 8 - 12
   d. 7 1/4 x 5

6. Rational numbers can be expressed as

   a. fractions.
   b. whole numbers.
   c. decimals.
   d. real numbers.

7. Which example illustrates the reflexive property of equality?

   a. 26 = a
   b. a = b
   c. a = a
   d. b = 26

8. When you write a statement such as a = 6, what axiom of equality allows you to change this to 6 = a?

   a. Commutative property
   b. Symmetric property
   c. Reflexive property
   d. Distributive property

9. What axiom of equality applies to this statement? If a = b and b = c, then a = c.

   a. Transitive property
   b. Commutative property
   c. Reflexive property
   d. Distributive property

10. The addition of 27 + 31 can be written 31 + 27 because of the ____________ property.

    a. commutative
    b. associative
    c. reflexive
    d. transitive

11. Which example illustrates the associative property for addition?

    a. 3 + 6 + 9 = 18
    b. 18 = 9 + 6 + 3
    c. 9 + 6 + 3 = 9 + 9
    d. (3 + 6) + 9 = (9 + 3) + 6

12. 7 x 9 = 9 x 7 illustrates the ______________ for multiplication.

    a. commutative property
    b. symmetric property
    c. distributive property
    d. transitive property

13. Which example illustrates the associative property for multiplication?

    a. 2 x 9 x 3 = 54
    b. 54 = 2 x 9 x 3
    c. 2 + 9 + 3 = 3 + 9 + 2
    d. (3 x 9) x 2 = 3 x (2 x 9)

14. The law that states multiplication can be distributed over addition is the ____________________ property.

    a. distributive
    b. commutative
    c. transitive
    d. symmetric

15. Which example best describes the distributive property?

    a. 5 x (2 + 3) = (5 x 2) + (5 x 3)
    b. 5 x (2 + 3)
    c. 10 x 15
    d. 25

Solve items 16 through 19 by using the zero and unity elements.

16. 10 x 16 ÷ 0 =

    a. Impossible
    b. 16
    c. 0
    d. 1/16

17. 7 ÷ 1 =

    a. 1
    b. 1/7
    c. 7
    d. 0

18. 58 x 0 =

a. 0
b. 58
c. -580
d. 580

19. 561 x 1 =

a. 56
b. 561
c. 562
d. 5611

Simplify items 20 through 29 by using the symbols of grouping.

20. (8 ÷ 2) + 7

____

21. 5 + [3 x 2]

____

22. 8 + (3 + 5)

____

23. 4(3 + 7)

____

24. 2 {6 + 5}

____

25. (8 + 3) + 5

____

26. 2(5 + 3)

____

27. [4 x 3] + [4 x 7]

____

28. 17 $\overline{15 \text{ x } 2}$

____

29. {17 + 3} 18

____

Solve items 30 through 38 by using the rules for order of operations.

30. (7 + 3 + 2) ÷ 3 + 1

____

31. (7 + 3 + 2) ÷ (3 + 1)

____

32. 5[7 + 9] ÷ 4 + 3

____

33. 7 + 3(5 - 1) ÷ 6

____

34. 30 - 3(7 - 2)

____

35. 27 + 10 - (11 + 3)

____

36. (2 x 3 - 12 ÷ 3) ÷ 2

____

37. 6(6 + 8) + 8 - 32 ÷ 16

____

38. (15 - 3 + 8 ÷ 2) ÷ 8 x 5

____

UNIT SUMMARY

This study unit has introduced or reintroduced you to the history of the real numbers. You have identified the natural numbers, integers, and the rational numbers within the real number system. You have identified the laws and properties that pertain to the number system. Try to remember these laws, their names, and how they are used with the basic operation of numbers. You can now recognize the symbols of grouping and solve expressions using the rules for order of operations.

Unit Exercise Solutions

		Reference
1.	d. Invention of zero	1101c
2.	a. cardinal and place values	1101c
3.	d. 3, 15, 27	1201
4.	b. 4 + 9 =	1202a
5.	c. 8 - 12	1301a
6.	a. fractions	1301b
7.	c. a = a	1401a
8.	b. Symmetric property	1401b
9.	a. Transitive property	1401c
10.	a. commutative	1402a
11.	d. (3 + 6) + 9 = (9 + 3) + 6	1402b
12.	a. commutative property	1402c
13.	d. (3 x 9) x 2 = 3 x (2 x 9)	1402d
14.	a. distributive	1403
15.	a. (5 x 2) + (5 x 3)	1403
16.	a. Impossible	1501a
17.	c. 7	1501b
18.	a. 0	1501a
19.	b. 561	1501b
20.	(8 ÷ 2) + 7 = 4 + 7 = 11	1601
21.	5 + [3 x 2] = 5 + 6 = 11	1601
22.	8 + (3 + 5) = 8 + 8 = 16	1601
23.	4(3 + 7) = 4(10) = 40	1601
24.	2{6 + 5} = 2{11} = 22	1601
25.	(8 + 3) + 5 = 11 + 5 = 16	1601
26.	2(5 + 3) = 2(8) = 16	1601
27.	[4 x 3] + [4 x 7] = 12 + 28 = 40	1601
28.	$17\ \overline{15 \times 2} = 17\ \overline{30} = 510$	1601
29.	{17 + 3} 18 = {20} 18 = 360	1601
30.	(7 + 3 + 2) ÷ 3 + 1 = 12 ÷ 3 + 1 = 4 + 1 = 5	1701a,b,c
31.	(7 + 3 + 2) ÷ (3 + 1) = 12 ÷ 4 = 3	1701a,b,c
32.	5[7 + 9] ÷ 4 + 3 = 5[16] ÷ 4 + 3 = 80 ÷ 4 + 3 = 20 + 3 = 23	1701a,b,c
33.	7 + 3(5 - 1) ÷ 6 = 7 + 3(4) ÷ 6 = 7 + 12 ÷ 6 = 7 + 2 = 9	1701a,b,c

	Reference
34. 30 - 3(7 - 2) =	
30 - 3(5) =	
30 - 15 =	
15	1701a,b,c
35. 27 + 10 - (11 + 3) =	
27 + 10 - 14 =	
37 - 14 =	
23	1701a,b,c
36. (2 x 3 - 12 ÷ 3) ÷ 2 =	
(6 - 4) ÷ 2 =	
2 ÷ 2 =	
1	1701a,b,c
37. 6(6 + 8) + 8 - 32 ÷ 16 =	
6(14) +8 - 32 ÷ 16 =	
84 + 8 - 32 ÷ 16 =	
84 + 8 - 2 =	
90	1701a,b,c
38. (15 - 3 + 8 ÷ 2) ÷ 8 x 5 =	
(15 - 3 + 4) ÷ 8 x 5 =	
(12 + 4) ÷ 8 x 5 =	
16 ÷ 8 x 5 =	
2 x 5 =	
10	1701a,b,c

STUDY UNIT 2

FRACTIONS AND PERCENTS

Introduction. One day our friend Charlie Caveman and his number one son Calvin were snooping and pooping through the boonies hunting for a tasty brontosaurus for the evening meal. Calvin spotted one and took up a position with good observation and field of fire while Charlie enveloped to the right. Upon arriving at the assault position, Charlie heard a clattering noise and saw Calvin had opened up the base of fire with his slingshot a little too early. Charlie, weary from his envelopment through the heavy brush, charged out and threw his M1 spear at the brontosaurus. His aim was not steady (poor sight picture) and he missed the beast. His spear hit a rock and broke in half. Charlie sat down to catch his breath while the bumbling Calvin went to retrieve the spear. He returned and said, "Dad, your spear broke. Here is the big half. The little half is splintered." Charlie's ears perked up at this unfamiliar terminology. He thought a moment and said, "Calvin, you chased away our evening meal, but you just invented fractions."

Of course this is fantasy, but it is used to show that even primitive man, although he primarily used his limited number of model groups, had an awareness that whole things had parts. Undoubtedly, things such as parts of a journey, portions of food, or parts of his herd were recognized for what they are, but it took a more sophisticated culture than Charlie's to actually invent fractions. The Egyptians and Romans had some symbolism to indicate the idea of fractions, but it was the Hindus who actually originated the system that we have today. They took good qualities from both the Egyptians and the Romans and added a vertical system of notation. The system allowed for variation in both the number parts (numerator) and the size of the parts (denominator). Remember that the numerator is the part above the line and the denominator is the part below the line.

This study unit will provide you with the skills you need to apply the basic principles of fractions and to perform operations with fractions, the decimal form of fractions, decimals, and percentages. As you are about to see, being able to perform these operations plays an important role in our daily lives as Marines.

Lesson 1. BASIC PRINCIPLES OF FRACTIONS

LEARNING OBJECTIVES

1. Given a series of fractions, use the proper operations to reduce them to the lowest terms.

2. Given a series of fractions, use the proper operations to reduce them to the higher terms.

2101. Reducing Fractions

The Golden Rule. The basic principle involving the manipulation of fractions is referred to by many textbooks as the "Golden Rule of Fractions." It is: If the numerator and denominator of a fraction are both multiplied or divided by the same number (other than zero), the value of the fraction is not changed. This principle is fundamental to all of the operations with fractions and there are two important points to be examined concerning it: First, the Golden Rule implies that you must remember to multiply or divide both terms of the fractions. If just the numerator is multiplied, for example, the resulting fraction will not be equal to the initial one. Second, this principle concerns multiplication and division only. If the same number is added to or subtracted from both the numerator and denominator, the value of the resulting fraction will not be the same as the initial one. The importance of this principle lies in the fact that it is often necessary to change the form of a fraction without changing its value. This change in form is known as a reduction. Strangely enough, the term reduction does not necessarily mean to make smaller, because in mathematics we can reduce to lowest terms or reduce to higher terms. The latter appears to be a contradiction, but this is the mathematical terminology. Remember, a reduction is a change in form without a change in value. Let's take a look at both terms.

a. Reducing to lowest terms. This is a familiar process that is accomplished by dividing each term of a fraction by a common factor; a process usually referred to as "cancellation." It has come along with the development of mathematics, but it does not accurately describe what is taking place during its use. Cancel implies to eliminate or get rid of, to leave nothing. Our mathematical cancel will always leave at least a value of 1. Let's look at some examples of reducing to lowest terms:

$$\frac{4}{6} = \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{6}}} = \frac{2}{3}$$

Note: (4 and 6 have the common factor 2) This middle step is usually mental, but it is inserted here to illustrate what is actually being done.

Reduce 15/30 to lowest terms. Suppose 5 is used as the common factor. In this case dividing each term by 5 gives us:

$$\frac{15}{30} = \frac{\overset{3}{\cancel{15}}}{\underset{6}{\cancel{30}}} = \frac{3}{6}$$

Here, though, 3/6 is not expressed in lowest terms; there is still a common factor 3:

$$\frac{3}{6} = \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}}} = \frac{1}{2}$$

The highest common factor, 15, would have given the quickest reduction, but sometimes the highest common factor is not readily evident. Always express an answer in lowest terms. When all common factors, other than 1, have been removed from the numerator and denominator, the fraction is in lowest terms.

b. Reducing to higher terms. This is the opposite of reducing to lowest terms. Instead of dividing by a common factor, multiply the numerator and denominator by some number greater than one. Reversing the procedure of the examples above:

$$\frac{2}{3} = \frac{2 \times 2}{2 \times 3} = \frac{4}{6}$$

$$\frac{1}{2} = \frac{15 \times 1}{15 \times 2} = \frac{15}{30}$$

Note: The middle step is usually mental.

It should be evident that to change a fraction to some desired higher term, you must divide the denominator of the smaller fraction into the denominator desired. This quotient is the common factor. For example, change 2/5 to an equivalent fraction with a denominator of 30.

$$\frac{2}{5} = \frac{?}{30}$$

What number must be multiplied times 5 in order to get 30? Since you have memorized the primary multiplication combinations, you know instantly that it is 6. Therefore multiply the numerator by 6 and get:

$$\frac{2}{5} = \frac{12}{30}$$

When the desired denominator reaches a size larger than a product of the primary combinations, then divide as mentioned above. As you know, the resulting fraction is the equivalent of the initial fraction although the form has changed. The ability to reduce to higher terms is essential for adding and subtracting fractions.

Lesson Summary. This lesson taught you how to reduce fractions to the lowest terms by dividing and to the highest terms by multiplying. Remember, reducing fractions (reduction) does not change the value of the fraction, only the form.

Lesson 2. OPERATIONS WITH FRACTIONS

LEARNING OBJECTIVES

1. Given a series of fractions, use the operations for addition of fractions to find the sum.

2. Given a series of fractions, use the operations for subtraction of fractions to find the difference.

3. Given a series of fractions, use the operations for multiplication of fractions to find the product.

4. Given a series of fractions, use the operations for division of fractions to find the quotient.

2201. Addition of Fractions

a. Adding fractions with like denominators. Do you remember from study unit 1 that the operation of addition of natural numbers is based on several principles, among them the commutative and associative laws and likeness? The addition operation with the rational numbers is also based on these same principles. Of particular concern is the principle of likeness which is controlled by the position of the numerals. In the natural numbers, recall that you add the numbers in the same columns. The column itself provides the likeness: first column is units, second column is tens, third is hundreds, etc. The determining factor of likeness in fractions is the size of the parts to be combined. Of course you know that the size of the part is the denominator. For example, the fractions 4/5 and 4/9, although both 4 of something, are not alike.

However the fractions 3/5 and 1/5 are alike because although the number of parts is different, the size of the parts is the same. When this is so, there is no difficulty in adding the fractions. The number of parts (numerators) are added to find the total number being considered. Symbolically you could characterize addition of fractions as: For any numbers a, b, and c (except when b = 0):

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$$

a practical example would be to add:

$$\frac{3}{12} + \frac{2}{12} + \frac{1}{12} = \frac{3 + 2 + 1}{12} = \frac{1}{2}$$

The middle step is not necessary. It is included to show the mental step that is taken. Note that the answer is expressed in lowest terms. Now, let's examine the operation of adding fractions with unlike denominators.

b. Adding fractions with unlike denominators. Take, for example, the addition of 1/3 and 5/6. Clearly, they are not alike. Something must be done to one or both of these fractions to make them alike. Remember that it was mentioned that the process of reduction would be necessary in adding and subtracting fractions. You want to change the form of either 1/3 or 5/6. It is easier to change the 1/3 to sixths without changing the value of the fraction. This is a problem similar to the ones in the last exercise.

$$\frac{1}{3} = \frac{?}{6}$$

Clearly the answer is 2/6. Now it is quite simple to add 2/6 + 5/6 = 7/6 = 1 1/6.

Again, note that the answer has been reduced to lowest terms. As a review, what kind of fraction is 7/6? How about 1 1/6? If you said improper and mixed number, you are correct. Let's try another example:

Add $\frac{1}{2}$ and $\frac{3}{8}$

First 1/2 should be changed to an equivalent fraction with a denominator of 8. This is 4/8 which can be added to 3/8.

$$\frac{4}{8} + \frac{3}{8} = \frac{7}{8}$$

How about when one denominator cannot be reduced to the size of the other one? What is done then? For example:

$$\frac{1}{5} + \frac{3}{4} = ?$$

The fraction 1/5 cannot be expressed as fourths and 3/4 cannot be expressed as fifths. As you know, a denominator must be found to which each of these denominators can be reduced (a common denominator). The quickest way to find a common denominator is to find the product of the denominators of the fractions to be added.

For example, the common denominator for 1/5 and 3/4 is 20. This simply means that each of these fractions can undergo a reduction to 20.

$$\frac{1}{5} = \frac{4}{20} \quad \text{while} \quad \frac{3}{4} = \frac{15}{20}$$

$$\text{therefore:} \quad \frac{1}{5} + \frac{3}{4} = \frac{4}{20} + \frac{15}{20} = \frac{19}{20}$$

What is a common denominator of 1/6 and 3/4? If you said 24, you are correct and the fractions would be reduced in this manner:

$$\frac{1}{6} + \frac{3}{4} = \frac{4}{24} + \frac{18}{24} = \frac{22}{24} = \frac{11}{12}$$

Notice that although 24 is a common denominator, there is a better one that could be used. Each of the denominators 6 and 4 can be reduced to 12. Remember this is called the least or lowest common denominator (LCD). See how the problem is worked using this:

$$\frac{1}{6} + \frac{3}{4} = \frac{2}{12} + \frac{9}{12} = \frac{11}{12}$$

The last step, reducing to lowest terms, was eliminated by selecting the LCD. Always try to find the LCD, but remember that the product of all of the denominators is a common denominator and can be used. The main disadvantage of this is that you will sometimes have to work with larger numbers and then reduce to lowest terms at the end. For those who like definitions and rules: The sum of two fractions with different denominators is obtained by replacing these fractions with equivalent fractions having a common denominator, and then adding the numerators. Stated symbolically, for any fractions:

$$\frac{a}{b} \text{ and } \frac{c}{d}, \quad \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

Note that throughout this study unit, addition has been performed in a horizontal form. It may just as well, and sometimes should, be done in a vertical form. Let's take a look at a couple of examples:

$$\begin{array}{rcr} 3\frac{2}{7} & = & 3\frac{4}{14} \\ +\ \frac{3}{14} & = & \frac{3}{14} \\ \hline & & 3\frac{7}{14} = 3\frac{1}{2} \end{array} \qquad \begin{array}{rcr} 9\frac{3}{4} & = & 9\frac{3}{4} \\ +\ 1\frac{1}{2} & = & 1\frac{2}{4} \\ \hline & & 10\frac{5}{4} = 11\frac{1}{4} \end{array}$$

As you can see by the two examples above, adding fractions in a vertical form rather than a horizontal form in most cases will be much easier. Let's now take a look at subtraction.

2202. Subtraction of Fractions

a. <u>Subtraction with like denominators</u>. As stated in lesson 1, subtraction is the opposite of addition. Since the rules of addition of natural numbers apply to rational numbers, it follows that the rules for subtraction of natural numbers apply to the subtraction of rational numbers. In particular, pay attention to the principle of likeness which states that only like fractions may be subtracted. This, as you know, means that only fractions having like denominators may be subtracted. This is done by subtracting the numerators and expressing the result over the common denominator, for example:

$$\frac{11}{15} - \frac{7}{15} = \frac{11 - 7}{15} = \frac{4}{15}$$

b. <u>Subtraction with unlike denominators</u>. For fractions with unlike denominators, the process again is similar to addition. Find the LCD and express each fraction as an equivalent fraction with the LCD. Subtract the numerators and reduce the resulting fraction to the lowest terms. For example:

$$\frac{7}{12} - \frac{1}{6} = \frac{7}{12} - \frac{2}{12} = \frac{5}{12}$$

$$\frac{7}{12} - \frac{3}{8} = \frac{14}{24} - \frac{9}{24} = \frac{5}{24}$$

c. <u>Subtraction with mixed numbers</u>. There is one other facet of subtraction that should be covered here, that is, subtraction of mixed numbers. At times the process of "borrowing" presents a problem to students. Let's look at several examples. First, fractions with like denominators, and no borrowing:

$$\begin{array}{r} 15\,\frac{2}{3} \\ -\ 6\,\frac{1}{3} \\ \hline 9\,\frac{1}{3} \end{array}$$

Here the numerators were subtracted and placed over the common denominator and then the integers were subtracted. Now, how about:

$$\begin{array}{r} 15\,\frac{1}{6} \\ -\ 12\,\frac{5}{6} \\ \hline \end{array}$$

Here the fractional part of the subtrahend (the number being subtracted) is larger than the fractional part of the minuend. This is solved by borrowing 1 from 15, converting it to its fractional form (1 = 6/6) and adding it to 1/6 (often done mentally):

$$\begin{array}{r} 15\,\frac{1}{6} \\ -\ 12\,\frac{5}{6} \\ \hline \end{array} = 14 + \frac{6}{6} + \frac{1}{6} = \begin{array}{r} 14\,\frac{7}{6} \\ -\ 12\,\frac{5}{6} \\ \hline 2\,\frac{2}{6} \end{array} = 2\,\frac{1}{3}$$

Let's take a look at another problem:

$$\begin{array}{rcccl} 4\frac{3}{8} & = & 4\frac{3}{8} & = & 3\frac{11}{8} \\ -\ 2\frac{3}{4} & = & 2\frac{6}{8} & = & -\ 2\frac{6}{8} \\ \hline & & & & 1\frac{5}{8} \end{array}$$

Note that 3/4 was converted to an equivalent fraction with the common denominator 8. This still left the fraction of the subtrahend smaller than that of the minuend. One was borrowed from 4 in the fractional form 8/8 and added to 3/8 totalling 11/8. This gave a workable fraction which could then be subtracted from in the normal manner. A definition of subtraction of fractions has been presented. For any two fractions it can be represented as:

$\frac{a}{b}$ and $\frac{c}{d}$, if ad > bc, then:

$$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

Note: the symbol > is read: is greater than.

Remember, only like fractions can be subtracted (fractions having like denominators), with unlike fractions, first, find the lowest common denominator, subtract, and reduce the result to the lowest terms.

2203. Multiplication of Fractions

Here again is an operation that is based on the same principles that applied to the natural numbers: the associative, commutative, and distributive laws. Multiplication for fractions is still a rapid or shortened addition process. All you need to do in order to multiply fractions, therefore, is draw on the principles you have already learned. There is one aspect of multiplication of fractions that is a unique process which proves troublesome to the thinking of some students. To multiply usually implies to gain, or to make larger. This is true with integers, but when dealing with multiplication by a fraction, the converse is true; a smaller quantity will result. For example if you take one-half of something, say a dozen eggs, you end up with a smaller quantity, 6 eggs, although you actually performed a multiplication, 1/2 times 12. You should keep this fact in mind when multiplying by a fraction and have confidence in the answers that are produced.

a. Multiplying a fraction by a whole number. Multiplication has been defined as a shortened addition of like addends. Therefore, multiplication of fractions is also a shortened addition of like addends. The addends in this case have like numerators and denominators. For example, to add:

$$\frac{2}{13} + \frac{2}{13} + \frac{2}{13} + \frac{2}{13}$$

In this case, 8/13 which you can see is 4 (the number of addends) times 2 (like numerator) over the like denominator. (Remember, the numerators are added and placed over the like denominator). Stated as a rule: To multiply a whole number times a fraction, multiply the whole number times the numerator and express this product as the numerator of a new fraction whose denominator is the same as the original fraction. Symbolically stated, for any numbers a, b, and c,

$$a \times \frac{b}{c} = \frac{ab}{c} \quad \text{Example:} \quad 7 \times \frac{3}{5} = \overset{*}{\frac{7 \times 3}{5}} = \frac{21}{5} = 4\,\frac{1}{5}$$

(* denotes mental step)

One further example by way of a pictorial representation (fig 2-1) should illustrate the rationale of multiplying an integer times a fraction: 5 x 3/4 = ?

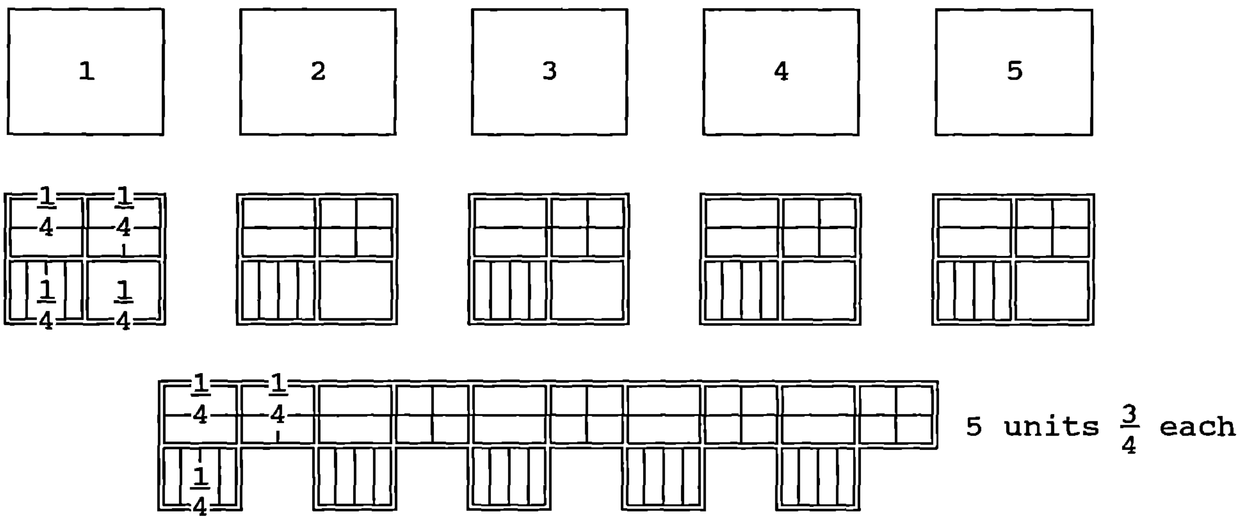


Fig 2-1. An integer times a fraction.

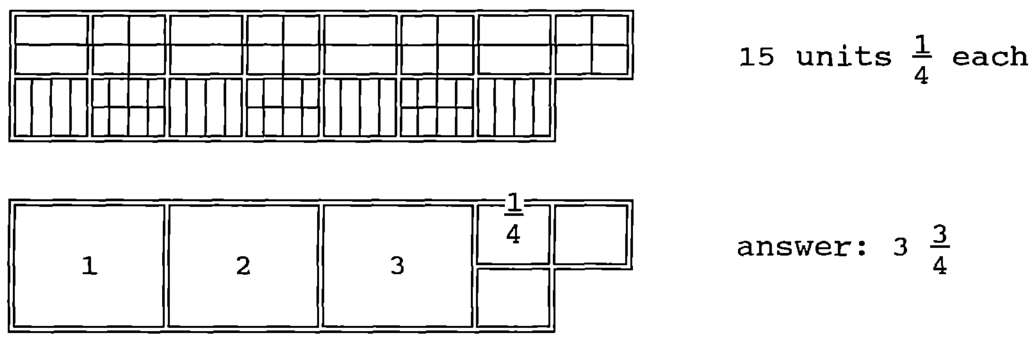


Fig 2-1. An integer times a fraction--cont'd.

This example (fig 2-1) is a pictorial representation that illustrates the rationale of multiplying an integer times (x) a fraction. It is sometimes referred to as a "putting-together" operation.

b. <u>Multiplying a whole number by a fraction</u>. Because of the commutative law, you know that this process will produce the same result as multiplying a fraction times an integer. The mechanics of the operation are the same: Multiply the numerator of the fraction times the integer and express this product as the numerator of a new fraction having the denominator of the original fraction. Symbolically stated, for any numbers a, b, and c, $\frac{a}{c} \times b = \frac{ab}{c}$. For example: Find $\frac{2}{5}$ of 8 acres (fig 2-2).

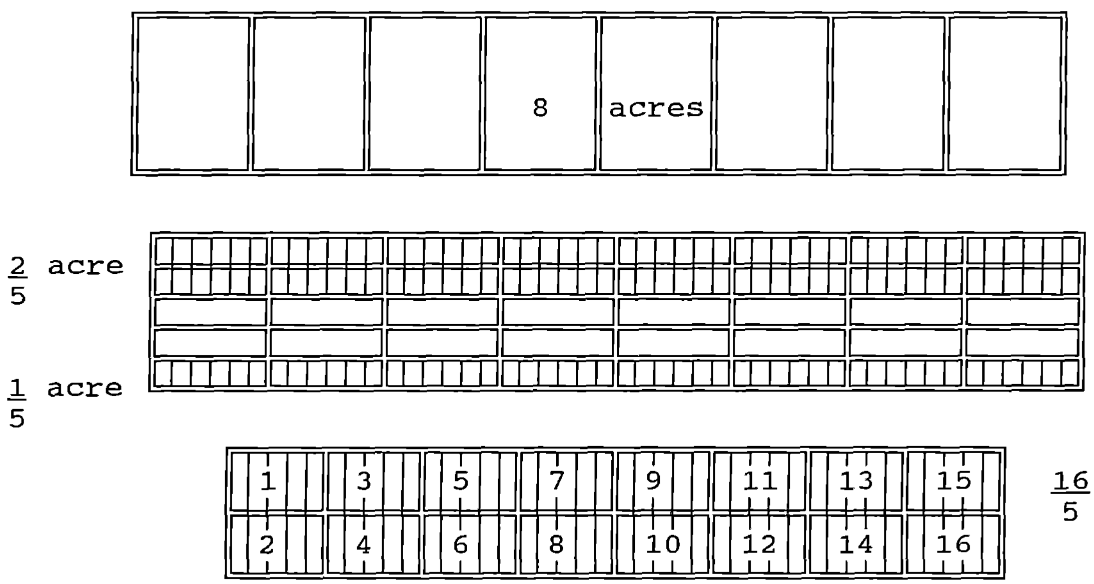
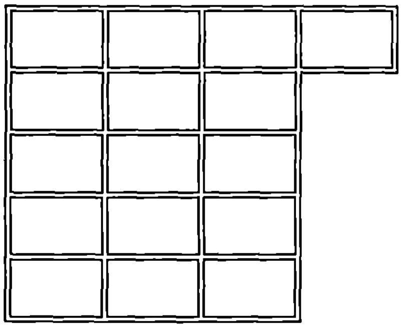


Fig 2-2. A fraction times an integer.



$\frac{2}{5} \times 8 = \frac{16}{5} = 3\,\frac{1}{5}$

Fig 2-2. A fraction times an integer--cont'd.

c. <u>Multiplying a fraction by a fraction (fig 2-3)</u>. In this operation you are interested in finding part of a part. The mechanics of the process are the same as the two already discussed except that the denominators are multiplied too. Symbolically stated, for any numbers a, b, c, and d (except when b and d = 0),

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Find $\frac{3}{4}$ of $\frac{3}{5}$ of an acre.

$$\frac{3}{4} \times \frac{3}{5} = ?$$

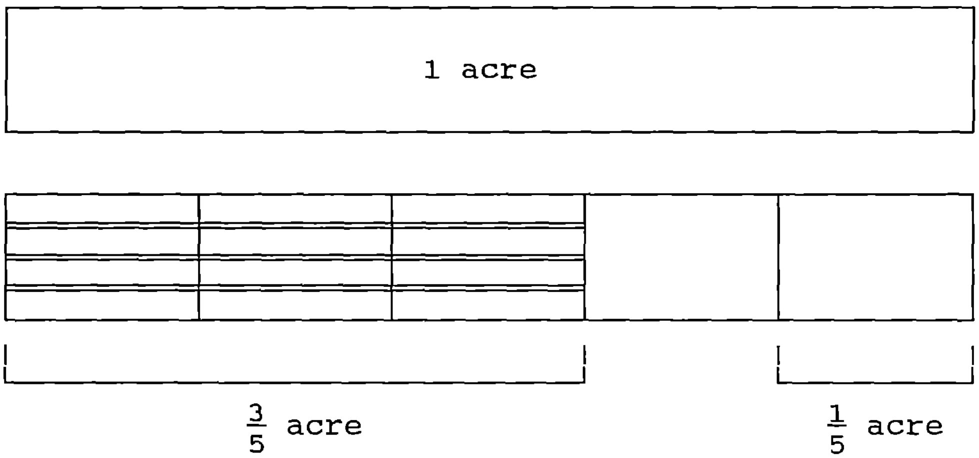
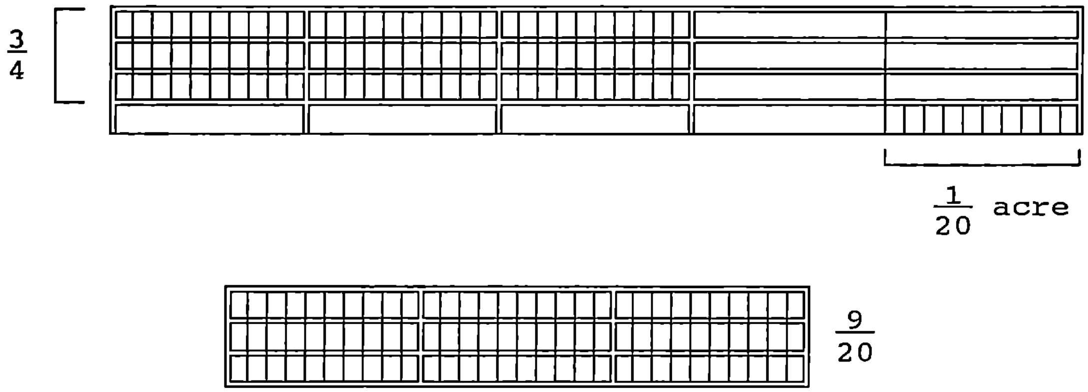


Fig 2-3. A fraction times a fraction.



$$\frac{3}{4} \times \frac{3}{5} = \frac{9}{20} \text{ of 1 acre}$$

Fig 2-3. A fraction times a fraction--cont'd.

d. Cancellation. Earlier in this lesson, the term cancellation was used to indicate reducing to lowest terms. Many times the proper use of canceling when multiplying fractions will enable you to work with smaller numbers and find answers more quickly. Look at the example worked first without canceling and then with canceling:

$$\frac{8}{18} \times \frac{6}{20} = ?$$

$$\frac{8}{18} \times \frac{6}{20} = \frac{48}{360} \quad \text{(This must now be reduced)}$$

$$\frac{48 \div 24}{360 \div 24} = \frac{2}{15}$$

$$\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{18}}} \times \frac{\overset{1}{\cancel{6}}}{\underset{5}{\cancel{20}}} = \frac{2}{15} \quad \text{(Divide 8 \& 20 by 4 and then divide 6 \& 18 by 6)}$$

Note: You may cancel the numerator of one fraction with the denominator of another when reducing the individual fraction. From this you should see that there is a definite advantage to canceling before multiplying. Let's see how canceling affects mixed numbers.

e. Multiplying with mixed numbers. This involves changing the mixed numbers to improper fractions and then proceeding with the rules for multiplying fractions. For example:

$$4\frac{2}{7} \times 5\frac{5}{9} = ?$$

$$\frac{\overset{10}{\cancel{30}}}{7} \times \frac{50}{\underset{3}{\cancel{9}}} = \frac{500}{21} = 23\frac{17}{21}$$

Note: First, change the mixed number to an improper fraction, cancel, then multiply. As you can see canceling with mixed numbers simplifies the equation.

2204. Division of Fractions

There are two computational techniques generally in use for division of fractions: The common denominator method and the simpler and probably more used inverted divisor method. Let's take a look at them.

a. Common denominator. In this method, a common denominator is first found just as in addition and subtraction. Each fraction is reduced to an equivalent fraction having the common denominator. The denominators are then disregarded and the numerators are divided. This can be shown with an example:

$$\frac{3}{4} \div \frac{1}{3} = ?$$

$$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12} \div \frac{1}{3} \times \frac{4}{4} = \frac{4}{12} = ?$$

The common denominator is 12. Reduce each fraction to an equivalent fraction with a denominator of 12.

$$\frac{9}{12} \div \frac{4}{12}$$

Divide 9 by 4 and 12 by 12.

$$\frac{9}{12} \div \frac{4}{12} = \frac{2\frac{1}{4}}{1} = 2\frac{1}{4}$$

You should see that the divided denominators will <u>always</u> be equal to 1, therefore this step may be eliminated, for example:

$$\frac{5}{16} \div \frac{5}{8} = ?$$

$$\frac{5}{16} \div \frac{5}{8} = \frac{5}{16} \div \frac{10}{16} = \frac{5}{10} = \frac{1}{2}$$

The common denominator method can also be done another way. This is by multiplying the original fractions by the LCD, for example:

$$\frac{3}{4} \div \frac{1}{3} = ?$$

$$\left(\overset{3}{\cancel{12}} \times \frac{3}{\underset{1}{\cancel{4}}}\right) \div \left(\overset{4}{\cancel{12}} \times \frac{1}{\underset{1}{\cancel{3}}}\right) = 9 \div 4 = 2\ \frac{1}{4}$$

Notice how LCD 12 cancels out denominators 4 and 3 making them 1. Using either method (division or multiplication) on fractions will provide you the quotient. Now let's see what happens when we invert the divisor.

b. <u>Inverted divisor</u>. This is the method that you probably learned in school. Simply stated, you remember it as invert the divisor and multiply. Let us see why this is done. This method is based on the idea of multiplying by the reciprocal of the <u>divisor</u>. Although this sounds complicated, remember from study unit 1 that the reciprocal of a number is 1 divided by the number, or, that the product of the number and its reciprocal is equal to 1. Let's do some examples to illustrate.

$$\frac{3}{4} \div \frac{1}{3} =$$

This example is read 3/4 divided by 1/3. What is the divisor? The divisor is the number by which a dividend is divided, in our case, it's 1/3. If 1/3 is the divisor, what is the reciprocal of the divisor? If you said 3/1, that's correct! Our next step is to multiply by the reciprocal of the divisor (multiply both the divisor and the dividend by this number).

$$\left(\frac{3}{4} \times \frac{3}{1}\right) \div \left(\frac{1}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{1}\right) = \frac{9}{4} \div 1 = \frac{9}{4} = 2\ \frac{1}{4}$$

Notice that the left fraction becomes 9/4 and the right fraction becomes 1. This will <u>always</u> be the case with the right fraction divisor. Therefore, the right half of the problem can be eliminated. You should see that the left fraction then fits the rule: Invert the divisor and multiply. Let's take a look at another example, first with the extra step and then without:

$$\frac{5}{16} \div \frac{5}{8} = \left(\frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{16}}} \times \frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{5}}}\right) \div \left(\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{8}}} \times \frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{5}}}\right) = \frac{1}{2} \div 1 = \frac{1}{2}$$

$$\frac{5}{16} \div \frac{5}{8} = \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{16}}} \times \frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{5}}} = \frac{1}{2}$$

This explanation should give you some insight into why the rule is valid and meaningful. There is one more example to discuss: dividing a fraction by an integer. Let's take a look at it.

When dividing a fraction by an integer the most important rule to remember is to invert the integer (divisor). For example:

$$\frac{7}{8} \div 2 =$$

$$\frac{7}{8} \div 2 = \frac{7}{8} \times \frac{1}{2} = \frac{7}{16}$$

Remember that 2 is actually the fraction 2/1 so the reciprocal is 1/2. Don't make the mistake of canceling the 2 with the 8.

<u>Lesson Summary</u>. This lesson provided you the necessary skill to use the operations for addition of fractions to find the sum, use the operations for subtraction of fractions to find the difference, use the operations for multiplication of fractions to find the product, and how to use the operations for division of fractions to find the quotient.

Lesson 3. DECIMAL FORM

1. Given decimal fractions, apply the operations to convert to common fractions.

2. Given common fractions, apply the operations to convert to decimal fractions.

3. Given a series of decimals, apply the operations for addition of decimals to find the sum.

4. Given decimals, apply the operations for subtraction of decimals to find the difference.

5. Given decimals, apply the operations for multiplication of decimals to find the product.

6. Given decimals, apply the operations for division of decimals to find the quotient.

It may seem odd to some students that decimals are discussed with fractions. On examination, however, you will see that decimals are just another way of writing fractions. Decimal fractions can be converted to common fractions, and vice versa, with very little difficulty. Let's look at the decimal system.

2301. Decimal Numeration

Our decimal system (fig 2-4) as the name implies, is based on the powers of ten. Every number to the left of the decimal point is 10 times the size of its neighbor to the right. Every number to the right of the decimal point is 1/10 the size of its neighbor to the left. For example, the thousands column is 10 times the size of the hundreds column. The tens column is 10 times the size of the units column. The first column to the right of decimal point is 1/10 of the units column or tenths. The next one to the right is 1/10 of the tenths or one-hundredths. The next one is thousandths, and so on. Note that figure 2-4 is only partial since the scale extends indefinitely in both directions. Also note that the system is symmetrical about the units column. That is, one to the left is <u>tens</u>, one to the right is <u>tenths</u>; two to the left is <u>hundreds</u>, two to the right is <u>hundredths</u>; six to the left is <u>millions</u>, six to the right is <u>millionths</u>. It is important to note that the system is not symmetrical about the decimal point. The primary purpose of the decimal point is to separate the whole and fractional parts of a number and to designate the location of the units digit.

MILLIONS	HUNDRED THOUSANDS	TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	UNITS	TENTHS	HUNDREDTHS	THOUSANDTHS	TEN THOUSANDTHS	HUNDRED THOUSANDTHS	MILLIONTHS
10^6	10^5	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}

Fig 2-4. Decimal numeration.

A decimal fraction, by definition, is a fraction whose denominator is a power of ten. There will always be a particular number of tenths, hundredths, thousandths, etc, and never any thirds, eighths, twelfths, etc. Other than having these fixed denominators, the only difference between common fractions and decimal fractions is the form in which they are written (fig 2-5). Also the last digit to the right in a decimal fraction determines the size of the fraction which is the same as the denominator.

$\frac{7}{10}$ = seven tenths = .7

$\frac{39}{100}$ = thirty-nine hundredths = .39

$\frac{421}{1000}$ = four-hundred twenty-one thousandths = .421

$\frac{57}{10000}$ = fifty-seven ten-thousandths = .0057

Fig 2-5. Comparison of form for common fractions and decimal fractions.

Notice that zeros are needed for their use as place holders with .0057. Remember, the only difference between common fractions and decimals is the form in which they are written. With this in mind, let's learn how to convert decimals to common fractions.

2302. Converting Decimals and Common Fractions

Since decimal fractions and common fractions are simply different forms of the same thing, it should come to mind that one can be changed to the other.

a. Decimal fraction to common fraction. We have already discussed the fact that a decimal fraction has a denominator of some power of ten. For example .8 is the same as 8/10 which can be reduced to 4/5. Some fractions can be reduced while others, like .33 = 33/100, cannot. It is relatively easy to determine by inspection whether a fraction can be reduced further. Since the denominators are all powers of ten, you need only try to divide by 2, 5, or 10 (factors of 10). If the fraction is not divisible by one of these, it is in its lowest terms. Let's take a look at some examples and see how this is applied:

$$.15 = \frac{15}{100} = \frac{3}{20} \qquad .88 = \frac{88}{100} = \frac{22}{25}$$

$$.008 = \frac{8}{1000} = \frac{1}{125} \qquad .0017 = \frac{17}{10000}$$

These examples show how easy it is to convert a decimal fraction to a common fraction. Now let's learn how to convert a common fraction to a decimal fraction.

b. Common fraction to decimal fraction. This operation is a little more complicated than the one just mentioned. It involves converting the denominator to a power of ten. For example, 9/25 can easily be converted. What is the most likely power of ten to which 25 could be converted? Did you say 100? Right. Now, what must be done to 25 to convert it to 100? You're right. Multiply it by 4. Don't forget the Golden Rule. If the denominator is multiplied by 4, then the numerator must also be multiplied by 4. Let's look at some examples:

$$\frac{9}{25} = \frac{4 \times 9}{4 \times 25} = \frac{36}{100} = .36$$

$$\frac{11}{20} = \frac{5 \times 11}{5 \times 20} = \frac{55}{100} = .55$$

$$\frac{17}{125} = \frac{8 \times 17}{8 \times 125} = \frac{136}{1000} = .136$$

$$\frac{3}{5} = \frac{2 \times 3}{2 \times 5} = \frac{6}{10} = .6$$

It is hoped that you have noticed that it will not always be possible to multiply the fraction by a whole number to get a power of ten. For example, denominators of 8, 13, 22, and 15 are not factors of integral powers of ten. In cases such as these it is easier to use a different method. This involves dividing the numerator by the denominator. This is called division of decimals which will be covered under operations with decimals.

2303. Operations with Decimals

a. Addition of decimals. As with integers, the addition process with decimals is based on the principles presented in study unit 1. In particular, again only like things may be added: tenths to tenths, hundredths to hundredths, etc. When addends are lined up in this manner, the decimal points fall in a straight line.

```
   2.18
  34.35
   0.14
+  4.90
-------
  41.57
```

The columns are added in the usual order and the decimal point of the sum falls directly below the decimal points of the addends.

b. Subtraction of decimals. This operation also involves no new principles. Place values are again aligned which causes the decimal points to align too. This alignment is also maintained in the answer, for example:

```
  45.76        8.64000
- 31.87      -  .00437
-------      ---------
  13.89        8.63563
```

The columns are subtracted in the usual order and the decimal point of the difference falls below the decimal points of the subtrahend.

c. Multiplication of decimals. Two cases of multiplying decimals will be considered: general multiplication and multiplication by powers of ten.

(1) General. Multiplication of decimals can be explained in two ways. The first is more of a common sense type of explanation rather than a statement of principles. For example, if you are multiplying 3.2 x 7.6 you must realize that although there are two digits in each of these numbers, part of each is a fraction. The answer will be close to 21 which is the product of the whole numbers 3 and 7.

```
   7.6
 x 3.2
   152
  228
  24.32
```

Let's look at a couple more examples:

```
   4.5627          126.3
 x 10.37        x 12.791
   319389           1263
  136881           11367
 456270            8841
47.315199          2526
                  1263
                1615.5033
```

Note: The non-fractional parts of the numbers in the first example are 10 and 4. Therefore, you know that the product must be close to 40. As you can see, this method is all right as long as the numbers don't get too large. One thing can be observed from the examples. This is that the answer in each case has as many digits following the decimal point as the sum of the digits following the decimal points in the multiplier and multiplicand. In the example 126.3 x 12.791, the multiplicand 126.3 has one digit following the decimal point and the multiplier 12.791 has three for a total of four. Note that the answer also has four.

The other explanation of multiplying decimals is by converting the decimal fractions to common fractions. Let's look at some examples:

decimal fraction = .4 x .37 = common fraction $= \frac{4}{10} \times \frac{37}{100} = \frac{148}{1000} = .148$

decimal fraction = 4.316 x 3.4 = common fraction $= \frac{4316}{1000} \times \frac{34}{10} = \frac{146744}{10000} = 14.6744$

$$\text{decimal fraction} = .453 \times 78.2 = \text{common fraction} = \frac{453}{1000} \times \frac{782}{10} = \frac{354246}{10000} = 35.4246$$

<u>Note</u>: The key is that the number of digits following the decimal point in the answer is the sum of the digits following the decimal points in the multiplier and the multiplicand. This leads us to the general rule for multiplication of decimals: Ignore the decimal points and multiply as though the multiplier and multiplicand were integers. Then locate the decimal point in the product to the left as many places as there are to the right of the decimal points in the multiplier and multiplicand.

(2) <u>Power of ten</u>. This operation is used enough to warrant special comment. By doing enough examples, you can discover for yourself what happens to the decimal point when you multiply by 10, 100, 1000, etc., or by .1, .01, .001, etc.

10 x 7 = 70 1000 x .5891 = 589.1

100 x 94 = 9400 10,000 x 9.8556 = 98556

<u>Note</u>: When multiplying by a power of ten, the decimal point is moved to the <u>right</u> as many places as there are zeros in the multiplier. Another aspect of multiplying by powers of ten is multiplying by .1, .01, .001, etc., which are negative powers of ten. This operation is opposite in effect to the one just described.

.01 x 153 = 1.53 .0001 x 13.28 = .001328

.001 x 348.2 = .3482 .01 x .0177 = .000177

<u>Note</u>: The effect here is to move the decimal point to the <u>left</u> as many places as there are decimal places (not zeros) in the multiplier.

As mentioned, this operation occurs frequently and you can save time by recognizing the problem and moving the decimal point without multiplying. Practice this and be confident in it and you will save time in computations.

d. <u>Division of decimals</u>. Four things will be discussed: general division of decimals, division by powers of ten, reducing a common fraction to a decimal fraction, and rounding off. Let's take a look at them.

(1) <u>General division of decimals</u>. Here, as in the other discussions, the purpose is not to teach you an operation, but to give you some insight into why the decimal point is moved, or not moved, in dividing decimals. You learned earlier in this lesson that multiplication of a decimal by a power of ten has the effect of moving the decimal to the right. This in effect is what is done in division of decimals. If the divisor can be transformed into a whole number, the division can be accomplished in the same manner as division of integers. Two things are done without actually naming them as such. First, the divisor is actually being multiplied by some power of ten when the decimal is moved to the right to make it a whole number. Second, since the divisor and dividend can be considered as the numerator and denominator of a fraction, if the denominator (divisor) is multiplied by some number, then the numerator (dividend) must be multiplied by the same number in order not to change the value of the fraction. For example:

$$3.6\ \overline{)45.75}$$

To make 3.6 a whole number, the decimal point is moved one place to the right which is the effect of multiplying by 10. The same thing must be done to the dividend as was done to the divisor:

$$36.\overline{)457.5}$$

Various methods are used to show that the decimal point has been moved. The most common one is illustrated.

$$.153\ \overline{)7.62}$$

$$.153.\ \overline{)7.620.}$$

↓ → ↑ ↓ → ↑

Note that a zero was added to the dividend which means the same as multiplying by 1000. Remember also that the decimal point in the quotient (answer) will be directly above the new location of the decimal point in the dividend. The reason for this can be shown several ways. One is to recall the rules for multiplication of decimals. For example in this problem:

```
        .186
      ______
2.9. |.5.394
 ↓→↑  ↓→↑
```

To check the answer you must multiply the quotient times the divisor. This should equal the dividend. In this case .186 x 29 = 5.394. By working several examples, you can prove to yourself that the decimal point truly ends up in the correct place when it is placed directly over the one in the dividend.

(2) Powers of ten. Division by a power of ten is the inverse of multiplying by powers of 10. Where multiplying had an increasing or upgrading effect, division has a decreasing or downgrading effect. Consequently, division will cause the decimal point to "move" in the opposite direction or to the left.

Note several examples:

$217,000 \div 1000 = 217$ $\quad$ $586.37 \div 100 = 5.8637$

$32,000 \div 10 = 3200$ $\quad$ $19.79 \div 1000 = .01979$

The generalization for division by a power of ten is that it has the effect of moving the decimal point in the dividend a number of places to the left equal to the number of zeros in the divisor.

(3) Reducing a common fraction to a decimal fraction. Recall that earlier in the study unit it was mentioned that there was another way of reducing a common fraction to a decimal fraction. As mentioned, this method involved division of decimals. Now that this has been discussed, this much-used method may be covered. Since a common fraction is actually a division of the numerator by the denominator, this is what is done:

```
         .5                      .75
1/2 = 2 )1.0            3/4 = 4 )3.00
         1 0                     2 8
                                   20
                                   20

         .375                     .85
3/8 = 8 )3.000        17/20 = 20 )17.00
         2 4                      16 0
           60                      1 00
           56                      1 00
            40
            40
```

Each of these came out even (no remainder), but this is not always the case. In some instances, you may want to round off your answer to some particular decimal place.

(4) <u>Rounding off</u>. If your answer does not come out even (there is a remainder), you will then express your remainder as a fraction. In division of decimals, this expression of the remainder may be eliminated by rounding off the answer to some desired decimal place. Common practice is to carry the division to one degree of accuracy more than is desired and then round off. For example, if the answer to a problem is 17.237 and accuracy to hundredths is all that is desired, the 7 may be eliminated.

If the extra digit is 0, 1, 2, 3, or 4, it is usually dropped. If it is 5, 6, 7, 8, or 9, one is added to the digit to the left. Therefore, 17.237 rounded to hundredths would be 17.24. Some other examples are:

original answer	nearest hundredth	nearest tenth
36.42793	36.43	36.4
427.6439	427.64	427.6
.34625	.35	.4
1.4963	1.50	1.5

Remember the rule, 0,1,2,3, or 4 round down, and 5,6,7,8, or 9 round up.

Lesson Summary. This lesson provided you with the necessary skill to apply the operations for converting decimal fractions to common fractions, and common fractions to decimal fractions. Furthermore, you learned how to apply the operations of addition and subtraction of decimals. Lastly, you were able to apply the operations for multiplication and division of decimals.

Lesson 4. PERCENTAGE

LEARNING OBJECTIVES

1. Given a common fraction or a decimal, apply the operations to convert to percent.

2. Given a percent, apply the operations to convert to a common fraction or decimal.

3. Given a number, use the operations to find what percent this number is to another number.

4. Given a percent of a known percent, use the proper operations to find the number this percent represents.

5. Given a percent and the number it represents, use the operations for finding the unknown number.

2401. Converting Percents, Fractions, and Decimals

The word "percent" is derived from Latin. It was originally "per centum," which means "by the hundred." Thus, the statement is often made that percent means hundredths. Percentage deals with the groups of decimal fractions whose denominators are 100--that is, fractions of two decimal places. Since hundredths were used so often, the decimal point was dropped and the symbol % was placed after the number and read "percent" (per 100). Thus, 0.15 is read "15 hundredths," and 15% is read "15 percent." Both represent the same value, 15/100 or 15 parts out of 100.

a. Changing decimals to percents. To change a decimal to a percent, the first step is to move the decimal point two places to the right. After moving the decimal point two places to the right, add the percent sign (%), for example:

$$.35 = 35\%$$

Remember, a decimal point is not shown with whole percents; 63% does not have a decimal point shown. Mentally, however, a decimal point is placed to the right of the number three (63.%).

Let's look at another example:

$$.83 = 83\%, \text{ not } 83.\%$$

However, as seen in the next example, fractional percents do have decimal points.

$$56\frac{1}{2}\% = 56.5\%$$

Now that you know how to change decimals to percents, let's learn how to change fractions to percents.

b. Changing fractions to percents. To change a fraction to a percent, first change the fraction to a decimal and then the decimal to a percent, for example:

$$\frac{3}{8} = .375 = 37.5\%$$

Let's try another example:

$$\frac{3}{4} = .75 = 75\%$$

Remember, change the fraction to a decimal and then change the decimal to a percent. Up to this point we have learned how to change decimals and fractions to percents; now, let's learn how to change percents back to decimals and fractions.

c. Changing percents to decimals. Since you do not compute with numbers in the percent form, it is often necessary to change a percent back to its decimal form. The steps are just the opposite of that used in changing decimals to percent. You change a percent to a decimal by dropping the percent sign and moving the decimal point two places to the left. For example, 12.5% becomes 12.5 after dropping the percent sign. Then you move the decimal point two places to the left and 12.5 becomes .125. Let's look at a few more examples:

$$34\% = .34$$

$$24.5\% = .245$$

$$125\% = 1.25$$

When a fractional percent (3/4%, 2/34%, 9/16%, etc.) is to be changed to a decimal, you must first change the fractional percent to its decimal equivalent percent (.75%, .059%, .56%) before you drop the percent sign and then move the decimal point two places to the left (.0075, .00059, .0056), for example:

$$5\,\frac{3\%}{4} = 5.75\% = .0575$$

$$\frac{1\%}{2} = .5\% = .005$$

Now that you have learned how to change percents to decimals, let's learn how to change percents to fractions.

d. Changing percents to fractions. To change a percent to a common fraction, first change the percent to a decimal. The second step is to change the decimal to a fraction and reduce to its lowest terms. For example:

$$80\% = .80 = \frac{80}{100} = \frac{4}{5}$$

$$65\% = .65 = \frac{65}{100} = \frac{13}{20}$$

$$22.5\% = .225 = \frac{225}{1000} = \frac{9}{40}$$

As you can see, changing decimals and fractions to percents is quite easy, but what happens if you need to know the percent of one number to another? Let's take a look and see.

e. Percent of one number to another number. You are now going to learn how to solve percentage problems. All percentage problems can be solved by substituting given information into the formula:

$$\frac{\text{small number}}{\text{large number}} = \frac{\_\_\_\%}{100\%}$$

Percentage numbers are all placed on one side of the equal sign in the formula and all other information on the other side.

(1) Finding what percent one number is to another. For example, in the problem: There are 25 Marines in the formation. Of the 25, 5 are NCO's. What percent of the Marines are NCO's? Substitute all known information into the formula.

Step #1 $\frac{\text{small}}{\text{large}} \frac{\%}{100\%} = \%$

Step #2 $\frac{5}{25} = \frac{x(\text{unknown})\%}{100\%}$

Step #3 $\frac{5}{25} = \frac{x}{1.00}$ (Change percents to decimals and cross multiply)

Step #4 $25x = 5$

Although this is actually an equation, and you will not be discussing algebra until the next study unit, you will cover enough algebra to complete the formula.

Note: The Golden Rule for the multiplicative property states that if two equal numbers are multiplied by the same number, the products are equal. The division property states that if two equal numbers are divided by the same number, the quotients will be equal. Just as addition and subtraction are inverse operations, so are multiplication and division. One operation can undo the other. This rule also applies to solving equations involving the multiplication and division properties of equality.

$$25x = 5$$

You are looking for some number that when multiplied by 25 will give a product of 5.
The objective in solving an equation is to isolate the variable, X in this case. Here you have multiplication, so the inverse division operation will be applied.

Step #5 $\frac{25x}{25} = \frac{5}{25}$ (Note that $\frac{25x}{25} = 1x$. It is customary to drop the 1 since x and 1x are the same)

$x = \frac{5}{25}$

Step #6 $x = .20$

$x = 20\%$

Let's look at another example: Of the 60 meals picked up by the "platoon guide," 12 of them were turkey loaf. What percent were turkey loaf?

Step #1 $\frac{\text{small}}{\text{large}} = \frac{x\%}{100}$

Step #2 $\frac{12}{60} = \frac{x\%}{100\%}$ (Put known information into the formula)

Step #3 $\frac{12}{60} = \frac{x}{1.00}$ (Change percents to decimals and cross multiply)

Step #4 $60x = 12$

Step #5 $\frac{60x}{60} = \frac{12}{60}$

$x = \frac{12}{60}$

Step #6 $x = .20$ (Change decimal to percent)

$x = 20\%$

If you apply the steps involved, you will have no problem finding what percent one number is to another. You may be asking yourself, why is this important? This is very important to Marines. Percentages are used every day to help account for our men, material, equipment etc. However there is still more to learn. You must also be able to find the number that a percent represents and that is what we are going to learn next.

(2) Finding a percent of a number. This is probably the most used of all percentage problems. A typical problem would be: Twenty Marines work in the S-1 office. Ten percent can go on leave. What is the number of Marines that can go on leave? Here, as in the preceding examples, the known information can be put into the formula:

Step #1 $\frac{\text{small}}{\text{large}} = \frac{x\ \%}{100\%}$

Step #2 $\frac{x\text{(unknown)}}{20} = \frac{10\%}{100\%}$

Step #3 $\frac{x}{20} = \frac{.10}{1.00}$ (Change percents to decimals and cross multiply)

Step #4 $1x = 20 \times .10 = 2.0$

Step #5 $x = 2$ Marines can go on leave

Let's try one more example: During the year (365 days), elements of the 1st platoon were on patrol 48 percent of the time. How many days were they on patrol?

Step #1 $\frac{\text{small number}}{\text{large number}} = \frac{\%}{100\%}$

Step #2 $\frac{x\text{(unknown)}}{365} = \frac{48\%}{100\%}$

Step #3 $\frac{x}{365} = \frac{.48}{1.00}$ (change percents to decimals and cross multiply)

Step #4 $1x = 365 \times .48$

Step #5 $x = 175.2$ days on patrol

You have been taught how to find what percent one number is to another and how to find the number that a percent represents. You have one more area to learn. You must be able to find the unknown number if you are given a percent. Let's take a look and see how this is done.

(3) Finding the unknown number. A typical problem would be: Pvt Pile bought a shirt for $10.50. This was 35 percent of the money he had saved for clothing. How much had he saved? As in the two preceding examples, the problem can be solved by putting the known information into the formula:

Step #1 $\frac{\text{small number}}{\text{large number}} = \frac{\%}{100\%}$

Step #2 $\frac{10.50}{x\text{(unknown)}} = \frac{35\%}{100\%}$ (Change percents to decimals and cross multiply)

Step #3 $\frac{10.50}{x} = \frac{.35}{1.00}$

Step #4 $.35x = 10.50$

Note: Here you must use the inverse division operation (Step 5). You are looking for some number that when multiplied by .35 will give you a product of 10.50.

Step #5 $\frac{.35x}{.35} = \frac{10.50}{.35}$ (x = 10.50 ÷ .35)

Step #6 x = $30

Let's look at one more example: Cpl Slick spent $52 on liberty in Tokyo last Saturday. This was 65 percent of his bank roll. How much money did he have originally?

Step #1 $\frac{\text{small number}}{\text{large number}} = \frac{\_\_\%}{100\%}$

Step #2 $\frac{52}{x(\text{unknown})} = \frac{65\%}{100\%}$ (Change percents to decimals and cross multiply)

Step #3 $\frac{52}{x} = \frac{.65}{1.00}$

Step #4 .65x = 52

Step #5 $\frac{.65x}{.65} = \frac{52}{.65}$ (Use the inverse division operation: x = 52 ÷ .65)

Step #6 x = $80

Remember, the main factor in finding the unknown is the inverse division operation.

Lesson Summary. This lesson provided you with the necessary skill to apply the operations of percentage to convert a common fraction or decimal to a percent, to convert a percent to a common fraction or decimal, to find what percent one number is to another, to find the percent of a number, and to find the unknown number.

Unit Exercise: Complete items 1 through 74 by performing the action required. Check your responses against those listed at the end of this study unit.

Note: Complete items 1 through 6 by reducing each fraction to its lowest term.

1. $\frac{36}{48} = ?$

a. $\frac{3}{4}$ b. $\frac{4}{5}$ c. $\frac{3}{8}$ d. $\frac{6}{8}$

2. $\frac{28}{56} = ?$

a. $\frac{1}{5}$ b. $\frac{1}{4}$ c. $\frac{1}{3}$ d. $\frac{1}{2}$

3. $\frac{78}{102} = ?$

a. $\frac{15}{17}$ b. $\frac{13}{17}$ c. $\frac{15}{19}$ d. $\frac{13}{19}$

4. $\frac{99}{144} = ?$

a. $\frac{15}{16}$ b. $\frac{13}{16}$ c. $\frac{11}{16}$ d. $\frac{9}{16}$

5. $\frac{186}{378} = ?$

a. $\frac{29}{63}$ b. $\frac{31}{63}$ c. $\frac{29}{65}$ d. $\frac{31}{65}$

6. $\frac{680}{2520} = ?$

a. $\frac{17}{53}$ b. $\frac{19}{53}$ c. $\frac{17}{63}$ d. $\frac{19}{63}$

Complete items 7 through 12 by reducing each fraction to its higher term by supplying the missing numerator.

7. $\frac{4}{9} = \frac{?}{27}$

a. 12 b. 13 c. 16 d. 17

8. $\frac{3}{4} = \frac{?}{100}$

a. 15 b. 30 c. 60 d. 75

9. $\frac{7}{8} = \frac{?}{240}$

a. 110 b. 120 c. 210 d. 220

10. $\frac{5}{7} = \frac{?}{56}$

a. 30 b. 40 c. 42 d. 45

11. $\frac{5}{12} = \frac{?}{132}$

a. 55 b. 60 c. 66 d. 72

12. $\frac{9}{37} = \frac{?}{111}$

a. 18 b. 21 c. 27 d. 99

Complete items 13 through 40 by performing the action required.

13. $\frac{3}{8} + \frac{1}{3} = ?$

a. $\frac{1}{24}$ b. $\frac{8}{24}$ c. $\frac{9}{24}$ d. $\frac{17}{24}$

14.
$$\begin{array}{r} \frac{1}{9} \\ + \frac{1}{6} \\ \hline \end{array}$$

a. $\frac{5}{12}$ b. $\frac{5}{16}$ c. $\frac{5}{18}$ d. $\frac{5}{24}$

15.
$$\begin{array}{r} \frac{2}{7} \\ + \frac{1}{12} \\ \hline \end{array}$$

a. $\frac{24}{64}$ b. $\frac{31}{64}$ c. $\frac{24}{84}$ d. $\frac{31}{84}$

16.
$$\begin{array}{r} \frac{3}{8} \\ + \frac{7}{20} \\ \hline \end{array}$$

a. $\frac{26}{40}$ b. $\frac{29}{40}$ c. $\frac{26}{80}$ d. $\frac{29}{80}$

17. $\frac{3}{8}$ + $\frac{1}{12}$ + $\frac{2}{15}$ + $\frac{3}{20}$

$$\begin{array}{r} \frac{3}{8} \\ \frac{1}{12} \\ \frac{2}{15} \\ + \frac{3}{20} \\ \hline \end{array}$$

a. $\frac{69}{120}$ b. $\frac{89}{120}$ c. $\frac{89}{120}$ d. $\frac{102}{120}$

18.

$$\begin{array}{r} 7\frac{7}{8} \\ 2\frac{1}{3} \\ + 3\frac{5}{6} \\ \hline \end{array}$$

a. $12\frac{1}{24}$ b. $13\frac{1}{24}$ c. $14\frac{1}{24}$ d. $14\frac{9}{24}$

19. $\frac{1}{6} + \frac{1}{12} + \frac{1}{9} + \frac{1}{8}$

a. $\frac{34}{72}$ b. $\frac{35}{72}$ c. $\frac{36}{74}$ d. $\frac{37}{74}$

20. $3\frac{3}{7} + 4\frac{11}{63} + 10\frac{4}{9}$

a. $16\frac{1}{19}$ b. $16\frac{1}{21}$ c. $18\frac{1}{19}$ d. $18\frac{1}{21}$

21.

$$\begin{array}{r} \frac{7}{9} \\ - \frac{1}{6} \\ \hline \end{array}$$

a. $\frac{17}{18}$ b. $\frac{16}{18}$ c. $\frac{13}{18}$ d. $\frac{11}{18}$

22.

$$\begin{array}{r} 3\frac{1}{3} \\ - \frac{5}{6} \\ \hline \end{array}$$

a. $\frac{1}{4}$ b. $\frac{1}{2}$ c. $2\frac{1}{4}$ d. $2\frac{1}{2}$

23. $2\frac{3}{8} - \frac{5}{8}$

a. $1\frac{1}{4}$ b. $1\frac{3}{4}$ c. $2\frac{1}{4}$ d. $2\frac{3}{4}$

24. $3\frac{5}{36} - 1\frac{4}{9}$

a. $3\frac{25}{36}$ b. $3\frac{1}{4}$ c. $1\frac{25}{36}$ d. $1\frac{1}{4}$

25. $7\frac{5}{12} - 2\frac{7}{12}$

a. $3\frac{3}{5}$ b. $3\frac{5}{6}$ c. $4\frac{3}{5}$ d. $4\frac{5}{6}$

26. $3\frac{2}{3} - \frac{11}{12}$

a. $2\frac{2}{3}$ b. $2\frac{3}{4}$ c. $2\frac{7}{12}$ d. $2\frac{1}{2}$

27. $5 - 2\frac{2}{3}$

a. $2\frac{1}{3}$ b. $2\frac{2}{3}$ c. $2\frac{1}{6}$ d. $2\frac{5}{6}$

28. $2 - \frac{49}{64}$

a. $1\frac{21}{64}$ b. $1\frac{19}{64}$ c. $1\frac{17}{64}$ d. $1\frac{15}{64}$

29. $\frac{5}{8} \times 16$

a. 12 b. 11 c. 10 d. 9

30. $\frac{4}{5} \times 2$

a. $1\frac{3}{5}$ b. $1\frac{2}{5}$ c. $2\frac{3}{5}$ d. $2\frac{2}{5}$

31. $\frac{2}{3} \times \frac{5}{6}$

a. $\frac{7}{9}$ b. $\frac{5}{9}$ c. $\frac{9}{18}$ d. $\frac{7}{18}$

32. $6\frac{2}{3} \times 15$

a. 100 b. 103 c. 300 d. 303

33. $7\frac{1}{2} \times \frac{4}{15}$

a. 1 b. 2 c. 4 d. 6

34. $5 \times 17\frac{2}{3}$

a. $85\frac{2}{3}$ b. $86\frac{1}{3}$ c. $88\frac{1}{3}$ d. $89\frac{1}{3}$

35. $\frac{8}{17} \div \frac{3}{5}$

a. $\frac{24}{85}$ b. $\frac{28}{85}$ c. $\frac{38}{51}$ d. $\frac{40}{51}$

36. $\frac{6}{27} \div \frac{12}{35}$

a. $\frac{35}{54}$ b. $\frac{37}{54}$ c. $\frac{24}{35}$ d. $\frac{27}{35}$

37. $\frac{63}{360} \div \frac{28}{45}$

a. $\frac{11}{32}$ b. $\frac{9}{32}$ c. $\frac{147}{1350}$ d. $\frac{149}{1350}$

38. $6\frac{2}{3} \div 2\frac{3}{4}$

a. $1\frac{14}{33}$ b. $1\frac{15}{33}$ c. $2\frac{14}{33}$ d. $2\frac{15}{33}$

39. $114\frac{7}{12} \div 6\frac{7}{8}$

a. $14\frac{1}{3}$ b. $14\frac{2}{3}$ c. $16\frac{1}{3}$ d. $16\frac{2}{3}$

40. $(\frac{3}{7} - \frac{1}{14}) \div \frac{35}{49}$

a. $\frac{2}{3}$ b. $\frac{1}{2}$ c. $\frac{1}{4}$ d. $\frac{1}{8}$

Complete items 41 through 45 by reducing the decimal fractions to common fractions in their lowest terms.

41. .06

a. $\frac{3}{10}$ b. $\frac{6}{10}$ c. $\frac{3}{50}$ d. $\frac{3}{25}$

42. .278

a. $\frac{278}{500}$ b. $\frac{139}{500}$ c. $\frac{278}{750}$ d. $\frac{139}{750}$

43. .432

a. $\frac{216}{500}$ b. $\frac{108}{275}$ c. $\frac{64}{175}$ d. $\frac{54}{125}$

44. .00375

a. $\frac{3}{80}$ b. $\frac{3}{800}$ c. $\frac{5}{80}$ d. $\frac{5}{800}$

45. .8625

a. $\frac{69}{80}$ b. $\frac{75}{80}$ c. $\frac{5}{8}$ d. $\frac{7}{8}$

Complete items 46 through 49 by reducing each common fraction to a decimal fraction (power of ten).

46. $\frac{13}{50}$

a. .13 b. .16 c. .21 d. .26

47. $\frac{8}{25}$

a. .32 b. .24 c. .21 d. .16

48. $\frac{74}{125}$

a. .925 b. .592 c. .259 d. .425

49. $\frac{7}{20}$

a. .21 b. .27 c. .35 d. .37

Complete items 50 through 61 by performing the action required.

50.
```
   3.68
   4.975
   1.3
+ 16.42
```

a. 26.375 b. 25.375 c. 26.875 d. 25.875

51.
```
   64.2
     .04
   18.
 + 17.37
```

a. 97.61 b. 98.61 c. 99.61 d. 100.61

52.
```
     8.637
   492.
      .003
 +    .1
```

a. 400.740 b. 450.740 c. 499.740 d. 500.740

53.
```
   46.37
 - 18.48
```

a. 25.98 b. 25.89 c. 27.89 d. 27.98

54.
```
   21.300
 -  2.004
```

a. 19.692 b. 19.296 c. 18.692 d. 18.296

55.
```
   307.00
 -  60.42
```

a. 264.68 b. 256.58 c. 246.58 d. 244.68

56.
```
   46.37
 x 18.48
```

a. 856.9176 b. 856.9671 c. 865.9176 d. 865.9671

57.
```
   21.3
 x  2.004
```

a. 42.6852 b. 42.2586 c. 43.6852 d. 43.2586

58.
```
   307.
 x  60.42
```

a. 16548.94 b. 17548.94 c. 18548.94 d. 18458.94

59. $86.3\,\overline{)97.4200}$ Round to nearest hundredths (.00).

a. 1.12 b. 1.13 c. 1.21 d. 1.31

60. $.031 \overline{\smash{)}.497820}$ Round to nearest hundredths (.00).

a. 16.05 b. 16.06 c. 15.05 d. 15.06

61. $2.2 \overline{\smash{)}2.6390}$ Round to nearest hundredths (.00).

a. 1.20 b. 1.19 c. 1.91 d. 1.02

Complete items 62 through 65 by changing each item to a percent.

62. .12

a. .12% b. 1.2% c. 12% d. 120%

63. 1.25

a. 125% b. 12.5% c. 1.25% d. .125%

64. $\frac{1}{4}$

a. 14% b. 20% c. .25% d. 25%

65. $\frac{9}{40}$

a. .225% b. 2.25% c. 22.5% d. 225%

Complete items 66 through 68 by changing from percents to fractions (reduced to lowest terms).

66. 40%

a. $\frac{2}{6}$ b. $\frac{2}{5}$ c. $\frac{2}{4}$ d. $\frac{2}{3}$

67. 12.5%

a. $\frac{1}{5}$ b. $\frac{1}{6}$ c. $\frac{1}{7}$ d. $\frac{1}{8}$

68. 32%

a. $\frac{16}{25}$ b. $\frac{14}{25}$ c. $\frac{12}{25}$ d. $\frac{8}{25}$

Complete items 69 through 74 by performing the action required.

69. During an equipment inventory, Cpl Pyle found 14 E-Tools missing from the list of 60. What is his percent of missing E-Tools?

    a. 2.33% b. 23.3% c. 2.43% d. 24.3%

70. Four Marines out of thirty-two from 3rd platoon are on the rifle range. What percent are on the rifle range?

    a. 12.5% b. 1.25% c. 25% d. 2.5%

71. Of 40 Marines in the platoon, 80% are qualified swimmers. How many Marines are qualified swimmers?

    a. 23 b. 27 c. 29 d. 32

72. During the month of April (30 days), our company spent 23 percent of its time aboard ship. How many days did the company spend aboard ship? Round to the nearest day.

    a. 5 b. 6 c. 7 d. 8

73. Ten percent of the platoon are attached to Company A. If there are 4 Marines attached to the Company A, how many Marines are in the platoon?

    a. 40 b. 38 c. 30 d. 14

74. If <u>3/4</u> percent of the battalion are at sickbay (a total of 12 Marines), how many Marines are in the battalion?

    a. 1200 b. 1400 c. 1600 d. 1800

UNIT SUMMARY

This study unit provided you with the basic principles of fractions, the operations with fractions, decimal form and percentage. Study unit 3 will introduce you to algebra. You will then see how these basic principles are applied.

Unit Exercise Solutions

			Reference
1.	a.	$\frac{3}{4}$	2101
		$\frac{36}{48} = \frac{3}{4}$	
2.	d.	$\frac{1}{2}$	2101
		$\frac{28}{56} = \frac{1}{2}$	
3.	b.	$\frac{13}{17}$	2101
		$\frac{78}{102} = \frac{13}{17}$	
4.	c.	$\frac{11}{16}$	2101
		$\frac{99}{144} = \frac{11}{16}$	
5.	b.	$\frac{31}{63}$	2101
		$\frac{186}{378} = \frac{31}{63}$	
6.	c.	$\frac{17}{63}$	2101
		$\frac{680}{2520} = \frac{17}{63}$	
7.	a.	12	2101
		$\frac{4}{9} = \frac{12}{27}$	
8.	d.	75	2101
9.	c.	210	2101
10.	b.	40	2101
11.	a.	55	2101

		Reference
12. c.	27	2101
13. d.	$\frac{17}{24}$	2201
14. c.	$\frac{5}{18}$	2201
15. d.	$\frac{31}{84}$	2201
16. b.	$\frac{29}{40}$	2201

13. d. $\frac{17}{24}$

$$\frac{3}{8} = \frac{9}{24}$$
$$+\ \frac{1}{3} = \frac{8}{24}$$
$$\frac{17}{24}$$

14. c. $\frac{5}{18}$

$$\frac{1}{9} = \frac{2}{18}$$
$$+\ \frac{1}{6} = \frac{3}{18}$$
$$\frac{5}{18}$$

15. d. $\frac{31}{84}$

$$\frac{2}{7} = \frac{24}{84}$$
$$+\ \frac{1}{12} = \frac{7}{84}$$
$$\frac{31}{84}$$

16. b. $\frac{29}{40}$

$$\frac{3}{8} = \frac{15}{40}$$
$$+\ \frac{7}{20} = \frac{14}{40}$$
$$\frac{29}{40}$$

Reference

17. c. $\frac{89}{120}$ 2201

$$\begin{array}{rcl} \frac{3}{8} & = & \frac{45}{120} \\ \frac{1}{12} & = & \frac{10}{120} \\ \frac{2}{15} & = & \frac{16}{120} \\ + \frac{3}{20} & = & \frac{18}{120} \\ \hline & & \frac{89}{120} \end{array}$$

18. c. $14\ \frac{1}{24}$ 2201

$$\begin{array}{rcl} 7\ \frac{7}{8} & = & 7\ \frac{21}{24} \\ 2\ \frac{1}{3} & = & 2\ \frac{8}{24} \\ +\ 3\ \frac{5}{6} & = & 3\ \frac{20}{24} \\ \hline & & 12\ \frac{49}{24} = 14\ \frac{1}{24} \end{array}$$

19. b. $\frac{35}{72}$ 2201

$$\begin{array}{rcl} \frac{1}{6} & = & \frac{12}{72} \\ \frac{1}{12} & = & \frac{6}{72} \\ \frac{1}{9} & = & \frac{8}{72} \\ + \frac{1}{8} & = & \frac{9}{72} \\ \hline & & \frac{35}{72} \end{array}$$

Reference

20. d. $18\frac{1}{21}$ 2201

$$\begin{array}{rcr} 3\frac{3}{7} & = & 3\frac{27}{63} \\ 4\frac{11}{63} & = & 4\frac{11}{63} \\ +10\frac{4}{9} & = & 10\frac{28}{63} \\ \hline & & 17\frac{66}{63} = 18\frac{3}{63} = 18\frac{1}{21} \end{array}$$

21. d. $\frac{11}{18}$ 2202

$$\begin{array}{rcr} \frac{7}{9} & = & \frac{14}{18} \\ -\frac{1}{6} & = & \frac{3}{18} \\ \hline & & \frac{11}{18} \end{array}$$

22. d. $2\frac{1}{2}$ 2202

$$\begin{array}{rcr} 3\frac{1}{3} & = & 2\frac{8}{6} \\ -\frac{5}{6} & = & \frac{5}{6} \\ \hline & & 2\frac{3}{6} = 2\frac{1}{2} \end{array}$$

23. b. $1\frac{3}{4}$ 2202

$$\begin{array}{rcr} 2\frac{3}{8} & = & 1\frac{11}{8} \\ -\frac{5}{8} & = & \frac{5}{8} \\ \hline & & 1\frac{6}{8} = 1\frac{3}{4} \end{array}$$

Reference

24. c. $1\frac{25}{36}$ 2202

$$\begin{array}{rl} 3\frac{5}{36} = & 2\frac{41}{36} \\ = & \\ -1\frac{4}{9} = & 1\frac{16}{36} \\ \hline & 1\frac{25}{36} \end{array}$$

25. d. $4\frac{5}{6}$ 2202

$$\begin{array}{rl} 7\frac{5}{12} = & 6\frac{17}{12} \\ -2\frac{7}{12} = & 2\frac{7}{12} \\ \hline & 4\frac{10}{12} = 4\frac{5}{6} \end{array}$$

26. b. $2\frac{3}{4}$ 2202

$$\begin{array}{rl} 3\frac{2}{3} = & 2\frac{20}{12} \\ -\frac{11}{12} = & \frac{11}{12} \\ \hline & 2\frac{9}{12} = 2\frac{3}{4} \end{array}$$

27. a. $2\frac{1}{3}$ 2202

$$\begin{array}{rl} 5 = & 4\frac{3}{3} \\ -2\frac{2}{3} = & 2\frac{2}{3} \\ \hline & 2\frac{1}{3} \end{array}$$

Reference

28\. d. $1 \frac{15}{64}$ 2202

$$\begin{array}{rcr} 2 & = & 1\frac{64}{64} \\ -\frac{49}{64} & = & \frac{49}{64} \\ \hline & & 1\frac{15}{64} \end{array}$$

29\. c. 10 2203

$\frac{5}{8} \times 16 = \frac{80}{8} = 10$

30\. a. $1 \frac{3}{5}$ 2203

$\frac{4}{5} \times 2 = \frac{8}{5} = 1 \frac{3}{5}$

31\. b. $\frac{5}{9}$ 2203

$\frac{2}{3} \times \frac{5}{6} = \frac{10}{18} = \frac{5}{9}$

32\. a. 100 2203

$6 \frac{2}{3} \times 15 = \frac{20}{3} \times 15 = \frac{300}{3} = 100$

33\. b. 2 2203

$7 \frac{1}{2} \times \frac{4}{15} = \frac{15}{2} \times \frac{4}{15} = \frac{60}{30} = 2$

34\. c. $88 \frac{1}{3}$ 2203

$5 \times 17 \frac{2}{3} = 5 \times \frac{53}{3} = \frac{265}{3} = 88 \frac{1}{3}$

Reference

35. d. $\frac{40}{51}$ 2204

$$\frac{8}{17} \div \frac{3}{5} = \frac{8}{17} \times \frac{5}{3} = \frac{40}{51}$$

36. a. $\frac{35}{54}$ 2204

$$\frac{6}{27} \div \frac{12}{35} = \frac{\overset{1}{\cancel{6}}}{27} \times \frac{35}{\underset{2}{\cancel{12}}} = \frac{35}{54}$$

37. b. $\frac{9}{32}$ 2204

$$\frac{63}{360} \div \frac{28}{45} = \frac{\overset{9}{\cancel{63}}}{\underset{8}{\cancel{360}}} \times \frac{\overset{1}{\cancel{45}}}{\underset{4}{\cancel{28}}} = \frac{9}{32}$$

38. c. $2\ \frac{14}{33}$ 2204

$$6\ \frac{2}{3} \div 2\ \frac{3}{4} = \frac{20}{3} \times \frac{4}{11} = \frac{80}{33} = 2\ \frac{14}{33}$$

39. d. $16\ \frac{2}{3}$ 2204

$$114\ \frac{7}{12} \div 6\ \frac{7}{8} = \frac{\overset{25}{\cancel{1375}}}{\underset{3}{\cancel{12}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{55}}} = \frac{50}{3} = 16\ \frac{2}{3}$$

40. b. $\frac{1}{2}$ 2204

$$\left(\frac{3}{7} - \frac{1}{14}\right) \div \frac{35}{49} = \left(\frac{6}{14} - \frac{1}{14}\right) \div \frac{35}{49} = \frac{5}{14} \div \frac{35}{49} =$$

$$\frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{14}}} \times \frac{\overset{7}{\cancel{49}}}{\underset{7}{\cancel{35}}} = \frac{7}{14} = \frac{1}{2}$$

Reference

41. c. $\frac{3}{50}$ 2302

$.06 = \frac{6}{100} = \frac{3}{50}$

42. b. $\frac{139}{500}$ 2302

$.278 = \frac{278}{1000} = \frac{139}{500}$

43. d. $\frac{54}{125}$ 2302

$.432 = \frac{432}{1000} = \frac{54}{125}$

44. b. $\frac{3}{800}$ 2302

$.00375 = \frac{375}{100000} = \frac{3}{800}$

45. a. $\frac{69}{80}$ 2302

$.8625 = \frac{8625}{10000} = \frac{69}{80}$

46. d. .26 2302

$\frac{13}{50} = \frac{26}{100} = .26$

47. a. .32 2302

$\frac{8}{25} = \frac{32}{100} = .32$

48. b. .592 2302

$\frac{74}{125} = \frac{592}{1000} = .592$

Reference

49. c. .35 2302

$$\frac{7}{20} = \frac{35}{100} = .35$$

50. a. 26.375 2303

```
   3.68
   4.975
   1.3
+ 16.42
  26.375
```

51. c. 99.61 2303

```
   64.2
     .04
   18
 + 17.37
   99.61
```

52. d. 500.740 2303

```
    8.637
  492.
     .003
+    .1
  500.740
```

53. c. 27.89 2302

```
   46.37
 - 18.48
   27.89
```

54. b. 19.296 2303

```
   21.300
 -  2.004
   19.296
```

55. c. 246.58 2303

```
   307.00
 -  60.42
   246.58
```

Reference

56. a. 856.9176 — 2303

```
   46.37
 x 18.48
   37096
  18548
 37096
4637
 856.9176
```

57. a. 42.6852 — 2303

```
 21.3
x 2.004
    852
   000
  000
 426
 42.6852
```

58. c. 18548.94 — 2303

```
  307.
 x 60.42
    614
  1228
  000
1842
18548.94
```

59. b. 1.13 — 2303

```
            1.128 = 1.13
       ____________
86.3. |97.4.200
  ↓→↑     ↓→↑
       86 3
       11 1 2
         8 6 3
         2 4 90
         1 7 26
             7 640
             6 904
```

Reference

60. b. 16.06 2303

```
          16.058 = 16.06
       ┌─────────
.031.  │.497.820
↓ → ↑   ↓ → ↑
         31
         187
         186
           1 82
           1 55
             270
             248
```

61. a. 1.20 2303

```
        1.199 = 1.20
      ┌─────────
2.2.  │2.6.399
 ↓→↑    ↓→↑
      2 2
        4 3
        2 2
        2 19
        1 98
          210
          198
```

62. c. 12% 2401

.12 = 12%

63. a. 125% 2401

1.25 = 125%

64. d. 25% 2401

$\frac{1}{4}$ = 25%

65. c. 22.5% 2401

$\frac{9}{40}$ = 22.5%

Reference

66. b. $\frac{2}{5}$ 2401

$40\% = .40 = \frac{40}{100} = \frac{2}{5}$

67. d. $\frac{1}{8}$ 2401

$12.5\% = .125 = \frac{125}{1000} = \frac{1}{8}$

68. d. $\frac{8}{25}$ 2401

$32\% = .32 = \frac{32}{100} = \frac{8}{25}$

69. b. 23.3% 2401

$\frac{14}{60} = \frac{x\%}{100\%}$

$\frac{14}{60} = \frac{x}{1.00}$

$60x = 14$

$\frac{60x}{60} = \frac{14}{60}$

$x = .233$

$x = 23.3\%$ (missing E-Tools)

Reference

70. a. 12.5% 2401

$$\frac{4}{32} = \frac{x\%}{100\%}$$

$$\frac{4}{32} = \frac{x}{1.00}$$

$$32x = 4$$

$$\frac{32x}{32} = \frac{4}{32}$$

$$x = .125$$

x = 12.5% (are on the rifle range)

71. d. 32 2401

$$\frac{x}{40} = \frac{80\%}{100\%}$$

$$\frac{x}{40} = \frac{.80}{1.00}$$

$$1x = 32$$

x = 32 (qualified swimmers)

72. c. 7 2401

$$\frac{x}{30} = \frac{23\%}{100\%}$$

$$\frac{x}{30} = \frac{.23}{1.00}$$

$$1x = 6.90$$

x = 6.9 = 7 (days aboard ship)

Reference

73. a. 40 — 2401

$$\frac{4}{x} = \frac{10\%}{100\%}$$

$$\frac{4}{x} = \frac{.10}{1.00}$$

$$.10x = 4$$

$$\frac{.10x}{.10} = \frac{4}{.10}$$

x = 40 (Marines in the platoon)

74. c. 1600 — 2401

$$\frac{12}{x} = \frac{\frac{3}{4}\%}{100\%}$$

$$\frac{12}{x} = \frac{.0075}{1.00}$$

$$.0075x = 12$$

$$\frac{.0075x}{.0075} = \frac{12}{.0075}$$

x = 1600 (Marines in the battalion)

STUDY UNIT 3

ALGEBRA

Introduction. The beginnings of algebra as we know it date from around AD 830 when the Arabs and Hindus became an important part of our number history. The Arabs used algebra in their study of astronomy and it is believed that much of it was borrowed from the Hindus. The word algebra is derived from part of the title of a book written by an Arabian mathematician named Al-Khorwarazmi. The book, "ilm al-jabr wa'l magabalah," became known throughout Europe as "al-jabr" which you can see is similar to algebra. This study unit will enable you to identify and evaluate algebraic expressions, solve simple equations and inequalities, and solve problems involving proportions and percentages. Let's first look at algebraic expressions.

Lesson 1. ALGEBRAIC EXPRESSIONS

LEARNING OBJECTIVES

1. Given an algebraic expression, identify the monomial term.

2. Given an algebraic expression, identify the like term.

3. Given an algebraic expression, identify the polynomial term.

4. Given algebraic expressions, apply the proper operations to evaluate them.

3101. General Information

During this study of algebra you will be confronted with two kinds of numbers: The numerical symbols that you have been accustomed to such as 1, 2, 3, 4, 5, etc., and literal numbers, that is, numbers represented by letters of the alphabet. These literal numbers are usually the unknown quantity that you are to find. Since literal numbers can have any value and are not fixed, they are called variables; that is, the value of x, y, z, m, n, etc. can vary from problem to problem. On the other hand, 7 is 7, 6 is 6, 34 is 34, etc., no matter where they are used. The value of these number symbols (numerals) stays constant, and for this reason they are called constants. When a product involves a variable, it is customary to omit the multiplication symbol x so as not to confuse it with the literal number x. Thus, 3 times m is written as 3m and a times b is written ab. Recall that the product of two constants is not written in this manner, but is expressed by using the symbols of grouping that were discussed earlier such as: 6 x 4 is not written as 64 but as 6(4), or (6)(4).

Another customary practice in algebra is to use the raised dot to signify multiplication ($6 \cdot 4$, $a \cdot b$). Like the raised dot, there are additional terms you must become familiar with; let's look at them.

3102. Nomenclature

a. Algebraic expression. An algebraic expression is any combination of numerals, literal numbers, and symbols that represent some number. In its simplest form it is called a term. No part of a term is separated by addition or subtraction signs.

Examples: x and $173562ab^3c^2m^2p^4x^yy^2$ are both terms.

Note: Expressions, as just illustrated, can take a variety of forms.

b. Monomial. An expression of only one term is called a monomial. It can appear in several forms and can have several parts of its own or it may be as simple as a literal number, n. By adding one numeral to this literal number, for example, 5n, you have added new terminology. Five and n are factors of some product which, until you replace n with a constant, can only be known as 5n. Five is known as the coefficient of n. The literal number n is actually a coefficient of 5 also, but it has become conventional to refer to the numerical part of the term as the coefficient.

Example: In the term $176y^3$, 176 is the coefficient of y^3.

This example brings up the need to understand another item, exponent.

c. Exponent. This is the small raised number that shows the power of a number or how many times this number, called the base, is to be used as a factor.

Example: y^7 is y to the seventh power and indicates that y is to be a factor 7 times or $y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$.

Let's look at another example of a different form.

Example: The term $3x^4$ indicates that the 3 is the coefficient which is a factor of the term. The exponent 4 signifies that x, not 3x, is to be raised to the fourth power or used as a factor 4 times. Broken down, the term would appear as $3 \cdot x \cdot x \cdot x \cdot x$. Care must be taken so as not to associate the exponent with the wrong base.

Note: If this example were written: $(3x)^4$, which at first glance may seem the same, the base would be 3x. The parentheses indicate that the entire quantity 3x is to be used as a factor 4 times as opposed to the single quantity x mentioned previously. This example broken down would appear as $3 \cdot x \cdot 3 \cdot x \cdot 3 \cdot x \cdot 3 \cdot x$ or, rearranging with the cummutative principle for multiplication, $3 \cdot 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x$. The main thing to remember here is to carefully observe the way the equation is written prior to solving it.

Now that you have a basic knowledge of certain key terms, let's look at how likeness affects these terms.

3103. Property of Likeness

Earlier in the course, the property of likeness was used to perform operations with numbers. This same property applies to terms.
As you saw previously, terms can take many different forms and in order for them to be combined they must be the same.

Example: 15ab and 12765ab are similar. The same is true with $17abcdx^3$ and $abcdx^3$, $2y^4$ and $7y^4$, c^2 and $25c^2$. Two terms such as 8x and 3ab or x and x^2 are unlike and cannot be combined or simplified.

Let's now take a look at how the property of likeness is affected by addition, subtraction, multiplication, and division.

a. Addition and subtraction. Remember, every part of the literal portion of the number must be identical for the terms to be similar.

Example:	$10ab^2c - 4ab^2c = 6ab^2c$. If the expression contains like and unlike terms, only the like ones are combined: $8ab + 4x - 2ab = 6ab + 4x$.

b. Multiplication and division. Whereas adding and subtracting can be done only with like terms, any two terms can be multiplied or divided. We will confine ourselves, however, to like terms. The important difference with multiplication and division is that the letters (variables) are affected as well as the numbers (constants).

Note: Two important things to remember: When multiplying like terms, you add the exponents. When dividing like terms, you will have no letters (variables) in the quotient.

Example:	Multiply 7ac times 4ac. Each part of the two terms is multiplied $4 \cdot 7$, $a \cdot a$, and $c \cdot c$. The product is $28a^2c^2$.

Let's look at a different example using division.

Example:	Divide $10b^2$ by $5b^2$
	$\frac{10b^2}{5b^2} = 2$ Here we divide 10 by 5 and b^2 by b^2. With like terms, division of the letters (variables) will always be 1, therefore the answer is simply the quotient of the coefficients (2).

3104. Polynomials

There are two types of polynomials that are very common, but before we discuss these, let's review. Do you remember what the algebraic expression of only one term is called? If you said monomial, you are correct. An expression with only one term is called a monomial. Mono means one, therefore it's one term.

> Examples: $9m$, $-6y^3$, a, and 6.

Poly means more than one, so algebraic expressions with more than one term are called polymonials. The two most common polynomial's are binomial and trinomial. Let's take a look at them.

a. Binomial. A polynomial with exactly two terms is called a binomial.

> Examples: $-9x^4 + 9x^3$, $8m^2 + 6m$, $3m^5 - 9m^2$.

b. Trinomial. A polynomial with exactly three terms is called a trinomial.

> Examples: $9m^3 - 4m^2 + 6$, $19y^2 + 8y + 5$, $-3m^5 - 9m^2 + 2$.

Note: The determining factor of how many terms there are in an expression is the number of addition (+) and subtraction (-) symbols. Notice that the binomial example has two terms and only one addition/subtraction sign while the trinomial example has three terms and two addition/subtraction signs.

Now that you have a better understanding of the terms in algebraic expressions, let's apply the proper operations to evaluate them.

3105. Evaluation

Any algebraic expression can be evaluated to find out what number it represents if the values of the variables are known. It is a matter of arithmetic and following the rules of order of operations that were presented earlier. Your ability to evaluate expressions will be an aid to you in solving formulas and checking equations. The best way to evaluate expressions is to use these three steps: first, write the expression, second, substitute the given values, then do the arithmetic.

<u>Example:</u> If $p = 3$, evaluate $4p^2$

$4p^2 = 4(3)^2 = 4(9) = 36$ (Note that the 3 is placed in parentheses to indicate multiplication by 4, and that the 3 was squared before being multiplied by 4.)

Let's look at two more examples of a different form.

<u>Example 1</u>: If $m = 2$, evaluate $(8m)^2$

$(8m)^2 = (8 \cdot 2)^2 = (16)^2 = 256$

(Note that the entire quantity 8m is to be squared.)

<u>Example 2</u>: If $x = 3$, evaluate $2x^2 + 4x + 5$

$2x^2 + 4x + 5 = 2(3)^2 + 4(3) + 5 =$
$2(9) + 12 + 5 = 35$

<u>Note</u>: Although some of the steps can be combined and others done mentally, it is recommended that you write each step out and not take any short cuts.

<u>Note</u>: The most important thing to remember in evaluation is to follow the three steps: write the expression, substitute the given values, and then do the arithmetic.

<u>Lesson Summary</u>. During this lesson on algebraic expressions, you learned about general information, nomenclature, property of likeness, polynomials, and how to evaluate algebraic expressions. In your next lesson, you are going to combine these algebraic expressions to form algebraic sentences which will help you later in solving equations.

Lesson 2. ALGEBRAIC SENTENCES

LEARNING OBJECTIVES

1. Given a number of equations, evaluate if they are true or false.

2. Given a number of equations, use the proper operations to find the correct root.

3. Given a number of inequalities, evaluate if they are true or false.

3201. General Information

a. Definition. An algebraic sentence is a mathematical statement composed of algebraic expressions and one of these symbols: = (equals), ≠ (is not equal to), > (is greater than), < (is less than), ≥ (is greater than or equal to), and ≤ (is less than or equal to). Any sentence that uses the equal sign is an equation. Sentences that use the other signs are called inequalities.

b. Simple equations. As implied by the statement above, an equation is a statement that two things are equal. It may or may not be true.

> **Example:** 4 + 5 = 9 and 6 + 7 = 10 + 2 are equations. The first is true because 4 + 5 and 9 are symbols for the same number, but the second is false since 6 + 7 and 10 + 2 do not describe the same number.
>
> **Note:** Remember, statements such as 4 + 5 = 9 and 6 + 7 = 10 + 2, whether true or false, are called equations. Everything to the left of the equal sign is called the left member of the equation; everything to the right of the equal sign is the right member of the equation. The solution or answer to an equation is called the "root" of the equation.

Before you examine equations containing variables, let's use some arithmetic ability to determine whether some equations are true or false.

Examples:

Are the equations below true or false?

Equation #1

$9 - (1 + 8) = (9 - 1) + 8$

$9 - 9 = 8 + 8$

$0 = 16$ Obviously 0 does not equal 16. (False)

Equation #2

$$4(9 - 5) - \frac{18 + 12}{5} = 4 + 3 \cdot 2$$

$$4(4) - \frac{30}{5} = 4 + 6$$

$16 - 6 = 10$

$10 = 10$ (True)

Now that you have determined if an equation is true or false, let's learn how to find the correct root.

3202. Root

Do you remember what root means? That's right, a root is the solution or answer to an equation. You know how to solve an equation such as $x + 7 = 13$. It can be done by inspection, that is, simply by examining or instinctively knowing that the number to be added to 7 to obtain 13 is 6. This method can be used to solve many simple equations. Of course you realize that all equations cannot be solved by inspection; there must be a mathematical method. Let's look at these methods starting with addition and subtraction.

a. Addition and subtraction properties of equality. Can you remember that in study unit 1, some of the properties of numbers were discussed? By taking the properties of equality that were discussed and adding the addition and subtraction properties of equality, you obtain a process for solving equations. Let's look at an illustration.

Example: Two Marines receive equal basic pay:

Pfc. Hard = $845.10 and Pfc. Charger = $845.10.

Each one gets promoted to Lcpl.; a pay raise of $33.00:

Lcpl. Hard = $845.10 + $33.00

Lcpl. Charger = $845.10 + $33.00.

The new basic pay for both Marines = $878.10

Note: The basic pay has changed, but, Lcpl. Hard's pay is still equal to Lcpl. Charger's pay.

Note: This example of the addition property of equality shows that if the same number is added to equal numbers, the sums are equal. Symbolically, if $a = b$, then $a + c = b + c$. You should see that the subtraction property of equality could be proved in the same way. Symbolically, if $a = b$, then $a - c = b - c$. Let's see how these properties can be used to solve equations.

Look at these expressions: $100 - 7$, $n - 6$, $48 + 12$, and $x + 4$. Suppose you wanted to perform some operation that will preserve the quantity on the left. $100 - 7$ ____ $= 100$, $n - 6$ ____ $= n$, $48 + 12$ ____ $= 48$, and $x + 4$ ____ $= x$. What can be inserted in the blanks to produce the results on the right? In the first one, if 7 is subtracted from 100, what must be done to return to 100? The answer is to perform the opposite or inverse operation with the same number.

Example:

$$100 - \underline{7 + 7} = 100$$

What must be done to the example $n - 6$ to preserve the n? You must do the opposite or add 6.

Example:

$$n - \underline{6 + 6} = n$$

The other equations would look like this:

<u>Examples</u>:

$$48 + \underline{12 - 12} = 48$$

$$x + \underline{4 - 4} = x$$

You need one more item to enable you to solve equations using the addition and subtraction properties of equality. Do you remember the Golden Rule of Fractions? There is a similar one for equations. It states that whatever operation is performed on one <u>member</u> of an equation must also be performed on the other member. Let's use this principle to solve an equation.

<u>Example</u>:

$$x - 6 = 19$$

By first using the addition property, you can eliminate the 6 from the left member and leave the x by itself. This is the objective in solving any equation: to isolate the variable in one member of the equation. The left member of the equation looks like this:

<u>Example</u>:

$$x - 6 + 6 =$$

Since 6 was added to the left member, it must also be added to the right member.

<u>Example</u>:

$$x - 6 + 6 = 19 + 6$$

$$x = 25$$

This was a simple equation, one that could actually be solved by inspection; consequently, you probably knew that the answer was 25. Let's now check the answer to be sure that it is correct. The check is part of solving an equation. All you do is substitute your answer (25) for the variable (x) in the original equation.

Example:

Check: $x - 6 = 19$

$25 - 6 = 19$

$19 = 19$

The left member equals the right member making 25 the correct "root" for the equation. Let's look at some other equations.

Examples:

$y + 17 = 45$

$y + 17 - 17 = 45 - 17$

$y = 28$

Check:

$y + 17 = 45$

$28 + 17 = 45$

$45 = 45$

$x - .8 = 1.1$

$x - .8 + .8 = 1.1 + .8$

$x = 1.9$

Check:

$1.9 - .8 = 1.1$

$1.1 = 1.1$

In the next equation, the variable is on the right, but it is solved in the same manner.

<u>Example</u>:

$$314 = a + 165$$

$$314 - 165 = a + 165 - 165$$

$$149 = a$$

Check:

$$314 = 149 + 165$$

$$314 = 314$$

After solving several equations of this type you may feel that it is not necessary to write in the step involving the addition and subtraction property. Fine, this can be done mentally. However, it is suggested that if you are new to solving equations that you include this step until you have consistently solved each equation correctly and have confidence in your ability to add and subtract "in your head." Let's now look at multiplication and division.

b. <u>Multiplication and division properties of equality</u>. The properties are quite simple and to the point. The multiplicative property states that if two equal numbers are multiplied by the same number, the products are equal. Symbolically, if $a = b$, then $ac = bc$. The division property states that if two equal numbers are divided by the same number, the quotients will be equal. Symbolically, if $a = b$, then $a/c = b/c$. Just as addition and subtraction are inverse operations, so are multiplication and division. One operation can undo the other. The Golden Rule also applies to solving equations involving the multiplication and division properties of equality.

Example:

$$6x = 84$$

$$\frac{6x}{6} = \frac{84}{6}$$

$$x = 14$$

Check:

$$6(14) = 84$$

$$84 = 84$$

In the example above, you are looking for some number that, when multiplied by 6, will give a product of 84. Remember that the objective in solving an equation is to isolate the variable, in this case "x". From practice with addition and subtraction, you know that in each case the inverse operation was applied. Here you have multiplication, so the inverse operation of division is applied. Let's look at a few other equations.

<u>Examples</u>:

$$13m = 169$$

$$\frac{13m}{13} = \frac{169}{13}$$

$$m = 13$$

Check:

$$13(13) = 169$$

$$169 = 169$$

$$1.5n = 3$$

$$\frac{1.5n}{1.5} = \frac{3}{1.5}$$

$$n = 2$$

Check:

$$1.5(2) = 3$$

$$3 = 3$$

$$.7 = .7m$$

$$\frac{.7}{.7} = \frac{.7m}{.7}$$

$$m = 1$$

Check:

$$.7 = .7(1)$$

$$.7 = .7$$

In an equation involving division, the multiplication property will be used to find the root. For example, in n/4 = 6, you are looking for some number "n" that, when divided by 4, equals 6.

Example:

$$\frac{n}{4} = 6$$

$$4 \cdot \frac{n}{4} = 6 \cdot 4$$

$$n = 24$$

Check:

$$\frac{n}{4} = 6$$

$$\frac{24}{4} = 6$$

$$6 = 6$$

Note: Since n is divided by 4, multiply it by 4 to get 1 as the coefficient of n. Let's look at a couple more examples.

Examples:

$$\frac{c}{19} = 6$$

$$\frac{c}{19} \cdot 19 = 6 \cdot 19$$

$$c = 114$$

Check:

$$\frac{114}{19} = 6$$

$$6 = 6$$

$$\frac{a}{5} = .8$$

$$5 \cdot \frac{a}{5} = .8 \cdot 5$$

$$a = 4$$

Check:

$$\frac{4}{5} = .8$$

$$.8 = .8$$

Note: As with addition and subtraction, the intermediate step can be mental. You are encouraged to include it until you have confidence in yourself and consistently get correct answers.

c. Equations involving the four operative properties of equality. Often equations may involve a combination of properties such as addition and multiplication, addition and division, subtraction and multiplication, etc. The principles of solving the equations will remain the same; only the number of steps will change. The sequence in which to proceed is based on the principle of isolating the variable.

Example:

$$8y + 4 = 52$$

Here you have a multiplication (8y) and an addition (8y + 4). To isolate the variable (y), the first step is to eliminate the 4.

Example:

$$8y + 4 = 52$$

$$8y = 4 - 4 = 52 - 4$$

$$8y = 48$$

$$\frac{8y}{8} = \frac{48}{8}$$

$$y = 6$$

Check:

$$8y + 4 = 52$$

$$8(6) + 4 = 52$$

$$48 + 4 = 52$$

$$52 = 52$$

Let's look at a few more equations before we move on.

Example:

$$9a - 9 = 27$$

$$9a - 9 + 9 = 27 + 9$$

$$9a = 36$$

$$\frac{9a}{9} = \frac{36}{9}$$

$$a = 4$$

Check:

$$9a - 9 = 27$$

$$9(4) - 9 = 27$$

$$27 = 27$$

Example:

$2p - 5 = 8.5$

$2p - 5 + 5 = 8.5 + 5$

$2p = 13.5$

$$\frac{2p}{2} = \frac{13.5}{2}$$

$p = 6.75$

Check:

$2p - 5 = 8.5$

$2(6.75) - 5 = 8.5$

$13.50 - 5 = 8.5$

$8.5 = 8.5$

Example:

$7n - 3n = 40 + 16$

$4n = 56$

$$\frac{4n}{4} = \frac{56}{4}$$

$n = 14$

Check:

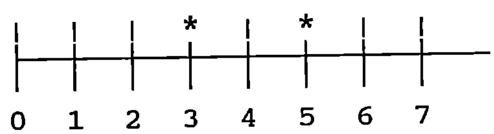
$7(14) - 3(14) = 40 + 60$

$98 - 42 = 56$

$56 = 56$

d. Inequalities. At the beginning of this lesson on algebraic sentences, you were given a group of symbols that denote inequalities. Years ago, inequalities were discussed only in college level math courses and sometimes in high school classes. But now, inequalities are introduced at the elementary school level. Why? To show the students comparisons between numbers. An elementary school child can readily interpret the statement 5 > 3 as 5 is greater than 3 and knows that this means the number 5 is larger or more than the number 3. He also associates these numbers on the number line and knows that 5 is to the right of 3.

Example:



At a more advanced level, inequalities may be applied to problems in economics, military tactics, industrial quality control, and research in general.

Problems concerning the most profitable division of time on TV programs, the allocation of production facilities in a factory to achieve maximum profit, or the determination of feed mixtures to minimize costs while satisfying standard food requirements are situations that represent a few of the areas to which the mathematics of inequalities can be applied. Most of this is beyond the realm of Math For Marines, but some of the aspects of inequalities are practical and may help you appreciate number relationships. Recall that an inequality is a statement that two quantities or expressions are not equal. Each expression is called a member, and one member may be < (less than), > (greater than), or ≠ (not equal to) the other member. Like an equation, an inequality may or may not be true.

Example:

$12 = 8 + (5-3)$ Is this true or false?

$12 = 8 + (5-3)$

$12 = 8 + 2$

$12 = 10$

This is false, 12 is not equal to 10.

Let's look at another example.

Example:

$11 < 8 + 2$

This is read 11 is less than 8 + 2?

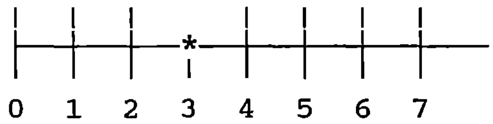
$11 < 8 + 2$

$11 < 10$

This is false, 11 is not less than 10.

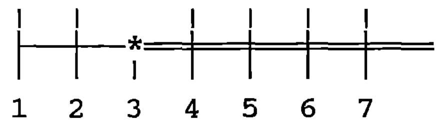
The next step is to extend these concepts to expressions that include variables. What does $x > 3$ mean? If you are talking about the "positive" whole numbers, this means all of the numbers to the right of 3 on the number line. Note that there is an asterisk above the 3 to show that it is not part of the solution.

Example:



However, if you are talking about the natural numbers, it would be the "counting" numbers to the right of 3.

Example:

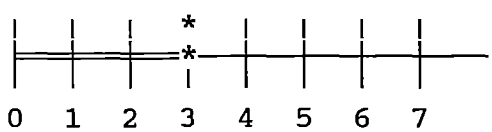


Do you remember the difference between whole and natural numbers? If you said zero (0) and negative numbers, your correct. Natural numbers do not include zero and negative numbers. That's the major difference between the two number lines. However, some inequalities are little more complicated and involve the same procedures used to solve equations.

> Example:
>
> $x + 2 \leq 5$
>
> $x + 2 - 2 \leq 5 - 2$
>
> $x \leq 3$

The graph would look like this for the "positive" whole numbers.

Example:



Note: The double asterisk above the 3 this time is there to show that 3 is part of the solution. This means that any number less than or equal to three is an answer to the inequality. In checking it, 3 or any of the numbers less than 3 can be substituted for the variable.

Let's look at a few more examples and determine if the inequalities are true or false.

<u>Example</u>:

$x + 2 \leq 5$ (Substitute 2 9/10 for x)

$2\ 9/10 + 2 \leq 5$

$4\ 9/10 \leq 5$

(This is true; $4\ 9/10 \leq 5$)

<u>Examples</u>:

$2x > 4$

$\frac{2x}{2} > \frac{4}{2}$

$x > 2$

(This is true, $x > 2$)

$3x + 4 > 13$

$3x + 4 - 4 > 13 - 4$

$3x > 9$

$\frac{3x}{3} > \frac{9}{3}$

$x > 3$

(This is true, $x > 3$)

If you are not sure how to determine whether inequalities are true or false review this section before moving forward.

<u>Lesson Summary</u>. During this lesson on algebraic sentences, you have learned how to determine if an equation is true or false, the proper operations to find the correct root, and how to determine if an inequality is true or false. Now that you know the basics, let's learn how to solve problems involving proportions and percentages.

Lesson 3. PROPORTION AND PERCENTAGE

LEARNING OBJECTIVES

1. Given problems involving proportions, solve using the operations for equations.

2. Given problems involving percentages, solve using the operations for equations.

3301. General Information

Problems involving both proportion and percentage are readily solved with the use of equations. Although percentage has been discussed previously, it is a topic that has enough use to warrant its being covered again from a different aspect; that of solution by equation. Proportion is another type of equation that can be used to solve many practical problems. Let's take a look at both of these.

3302. Proportion

During the training exercise, a platoon of Marines were clearing obstacles blocking their field of fire, and they had one lone tree left to be removed. The tree had to be dropped toward the east, but there was one small problem; a pipeline was located in the same direction. Cpl Chainsaw (the NCO in charge) didn't know the height of the tree, so he could not determine if the tree would hit the pipeline when it dropped. He knew that the pipeline was 70 feet from the tree. Cpl Chainsaw stared at the shadows cast by members of the platoon. He then whipped out his tape measure and began taking measurements of the Marines and their shadows. Lcpl Handsaw had a shadow length of 48" and his actual height was 72", Pfc Wedge had a shadow length of 42" and his actual height was 63", and Pvt Axe was holding a splitting maul which had a shadow length of 20" and its actual height was 30". Cpl Chainsaw double checked the height of the three objects and the length of the shadows he had measured and then scratched the following information in the dirt:

Object	Shadow length	Height
Lcpl Handsaw	48 in.	72 in.
Pfc Wedge	42 in.	63 in.
Splitting maul	20 in.	30 in.
Tree	40 ft.	?

Cpl Chainsaw thought for several minutes and then announced that the tree was 60 ft tall. Was he correct? Let's see how Cpl Chainsaw thought out his answer. First, he noticed that when he compared the shadow lengths to the actual heights, they were all proportional.

Lcpl Handsaw = $\frac{48}{72} = \frac{2}{3}$ Pfc Wedge = $\frac{42}{63} = \frac{2}{3}$ Splitting maul = $\frac{20}{30} = \frac{2}{3}$

This 2/3 is called a ratio. It is the comparison of two things by division, and is read 2 to 3, the fraction line being "to." An older, but still used form is 2:3 with the colon being read "to." Ratios are reduced to lowest terms as indicated by all of the above being reduced to 2/3. Cpl. Chainsaw assumed correctly that the principle would hold true for all objects measured at the same time and place.

He then had the problem:

$$\frac{40}{?} = \frac{2}{3}.$$

He needed to find a fraction with a numerator of 40 that is equivalent to 2/3. Do you remember how we solved this back in study unit 2? If you said "inverse division," you're correct. Let's take a look and find out:

<u>Example</u>:

$$\frac{40}{?} = \frac{2}{3}$$

2? = 40 x 3 = 2? = 120 (Cross multiply)

$\frac{2?}{2} = \frac{120}{2}$ (Divide)

? = 60

Yes, Cpl Chainsaw was correct. The correct answer is 60 ft. Lcpl Handsaw is now free to cut down the tree without hitting the pipeline because there is a distance of 10 feet to spare.

Referring back to our example and the definition of ratio, you see that a ratio compares two numbers. In the example, we had two sets of numbers, one set from the lengths of shadows and one set from the heights. All of the ratios were equal to one another. They are said to be <u>proportional</u>. A <u>proportion</u> is a statement of equality of two ratios.

Thus: $\frac{20}{30} = \frac{2}{3}$, $\frac{42}{63} = \frac{2}{3}$, $\frac{48}{72} = \frac{2}{3}$, and $\frac{40}{60} = \frac{2}{3}$ are all proportions.

The equal sign is read "as." The first one then is read 20 is to 30 as 2 is to 3. Another way of writing a proportion is with a double colon for the "as." 20:30::2:3. Let's now examine an important characteristic of proportions and how we can use the equation-solving techniques learned earlier to solve proportions.

$$\frac{2}{3} = \frac{6}{9}, \quad \frac{3}{4} = \frac{24}{32}, \quad \frac{5}{6} = \frac{15}{18}, \quad \frac{3}{8} = \frac{375}{1000}$$

In each of the above proportions, as in all proportions, there is an important relationship that exists between the two ratios.

Example:

$$\frac{2}{3} = \frac{6}{9}, \; 2 \cdot 9 = 3 \cdot 6, \; 18 = 18,$$

$$\frac{3}{4} = \frac{24}{32}, \; 3 \cdot 32 = 4 \cdot 24, \; 96 = 96,$$

$$\frac{5}{6} = \frac{15}{18}, \; 5 \cdot 18 = 6 \cdot 15, \; 90 = 90,$$

$$\frac{3}{8} = \frac{375}{1000}, \; 3 \cdot 1000 = 8 \cdot 375, \; 3000 = 3000.$$

If a proportion is true, the products of the numerator of one fraction and the denominator of the other fraction will be equal. This then can be useful in solving more difficult proportions where the outcome is not so obvious. For example, suppose Cpl. Chainsaw measured the shadow of another tree and found it to be 21 ft. How tall is the tree? You have then the length of the shadow (21) is to the height of the tree (x) as 2 is to 3.

Example:

$$\frac{21}{x} = \frac{2}{3}$$

From the characteristic just explained, you know the products of the numerator of one fraction and the denominator of the other will be equal.

Example:

$$\frac{21}{x} = \frac{2}{3}$$

$$2x = 21(3)$$

$$2x = 63$$

Now you have an equation of the form solved earlier in the study unit.

Example:

$$2x = 63$$

$$\frac{2x}{2} = \frac{63}{2}$$

$$x = 31\ 1/2$$

The height of the tree is 31 1/2 ft.

Let's look at another example of solving proportions.

Example:

The truck convoy took 4 hours to travel the first 75 miles of the trip. Assuming the average speed is the same for the remaining 60 miles, how much longer will it take to reach the objective?

Actually the proportion can be set up in several ways. If we say that the first ratio is 4 hours to 75 miles, 4/75, then the other would be unknown time, x, to 60 miles, x/60.

Example--cont'd:

$$\frac{4}{75} = \frac{x}{60}$$

$$75x = 240$$

$$\frac{75x}{75} = \frac{240}{75}$$

x = 3.2 hours or 3 hours and 12 minutes.

Note: Remember that .2 = 1/5, so 1/5 (60) = 12. You could do it another way by setting up ratios of the same units. The ratio of unknown time to known time is x/4. The ratio of distance for unknown time to distance for known time is 60/75.

$$\frac{x}{4} = \frac{60\ *}{75\ **}$$

$$75x = 240$$

$$x = 3.2$$

* Denotes distance at unknown time.
** Denotes distance at known time.

If you had a problem solving these examples, go back and review this section before moving on to percentages.

3303. Percentage

Earlier in the course you were introduced to percentages and you were given the methods to use for solving various percentage problems. Now that the principles of equation solving have been discussed, you will see how they can be applied to percentages. There are three types of percentage problems, sometimes referred to as cases involving: finding a percent of a number, finding what percent one number is of another, and finding a number when the percent of the number is known. It is a relatively simple matter to transform a sentence describing one of the percentage cases into an equation. The two key words in these sentences are "of" and "is." "Of" can be thought of as indicating multiplication and "is" can be replaced by an equal sign.

a. Finding a percent of a number. This is probably the most used of the three cases. A typical sentence would be: What is 35 percent of 14? Now, let's take this and make an equation out of it. "What" is the unknown (x), "is" will be replaced by the equal sign, the 35 percent is changed to a decimal, and "of" is replaced by a multiplication symbol.

Examples:

What is 35 percent of 14?

x = .35(14)

x = 4.9

Thus, 35 percent of 14 is 4.9.

What is 86 percent of 741?

x = .86(741)

x = 637.26

Thus, 86 percent of 741 is 637.26.

Now that you know how to find a percent of a number, let's find out what percent one number is of another.

b. Finding what percent one number is of another. A typical sentence would be: 20 is what percent of 80? Substitution of the symbols of the equation will be made in the same manner.

Example:

Twenty is what percent of 80?

20 = x%(80)

Now the equation says that 20 is equal to some unknown percent times 80. One slight adjustment to the equation will put it in a more familiar form. Using the commutative law for multiplication, change the positions of x percent and 80.

Example--cont'd:

$$20 = 80x \text{ (drop percent sign)}$$

$$\frac{20}{80} = \frac{80x}{80}$$

$$\frac{1}{4} = x \; *$$

$$.25 = x \; *$$

$$25\% = x$$

* These steps are shown but are not necessary.

Check:

$$20 = 25\%(80)$$

$$20 = .25(80)$$

$$20 = 20$$

Let's try another example. Forty-five Marines in the company are on the rifle range. There are one-hundred twenty Marines in the company. What percent of the company is on the range?

Example:

Forty-five is what percent of 120?

$$45 = x\%(120)$$

$$45 = 120x$$

$$\frac{45}{120} = x$$

$$.375 = x$$

$$37.5\% = x$$

Check:

$$45 = 37.5\%(120)$$

$$45 = .375(120)$$

$$45 = 45$$

Now that you know how to find what percent one number is of another, let's learn how to find a number when a percent of the number is known.

c. Finding a number when a percent of the number is known. A typical sentence would be: Forty is 2 percent of what number?

Example:

$$40 = 2\%(x)$$

$$40 = .02x$$

$$\frac{40}{.02} = \frac{.02x}{.02}$$

$$2000 = x$$

Check:

$$40 = 2\%(2000)$$

$$40 = .02(2000)$$

$$40 = 40$$

Let's take a look at another example. If 120 Marines (80%) in the company are deployed, what is the total number of Marines in the company?

Example:

120 is 80 percent of what number?

$120 = .80x$

$\frac{120}{.80} = x$

$150 = x$

Check:

$120 = 80\%(150)$

$120 = .80(150)$

$120 = 120$

What is the best way to simplify proportion and percentage word problems? If you said, "Write the statement in words and then substitute the words for mathematical symbols," you're correct. As you have seen from this study unit, writing and solving equations can greatly simplify the solution of percentage problems.

Lesson Summary. During this lesson you learned how to solve problems involving proportions and percentages using the operations for equations. It's now time to advance forward and attack the unit exercise.

Unit Exercise: Complete items 1 through 28 by performing the action required. Check your responses against those listed at the end of this study unit.

1. Which algebraic expression is a monomial term?

   a. $3x + y + x + y$
   b. $(xz)^3 - 1x^2$
   c. $100a - 35a$
   d. $45x^3y$

2. Which algebraic expression contains like terms?

a. $135abc^2$, $45abc$
b. $(xyz)^3$, $xy(z)^3$
c. $5a^2$, $(5a)^2$
d. $16xy$, $1xy$

3. Which algebraic expression is a polynomial term?

a. $2x - x + 4z^3$
b. $\frac{37xy}{1x - y}$
c. $\frac{6x + z}{xz}$
d. $46abc$

Complete items 4 through 10 by performing the proper operations of evaluation.

4. $8r^2$, $(r = 5)$

a. 80
b. 100
c. 200
d. 1600

5. $(8y)^2$, $(y = 5)$

a. 1600
b. 200
c. 100
d. 80

6. $a^3 - 2a^2 + a + 4$, $(a = 5)$

a. 584
b. 534
c. 84
d. 29

7. $\frac{x^2 + z^2}{y^2}$, $(x = 5,\ y = 2,\ z = 3)$

a. 4
b. 7
c. 3 1/2
d. 8 1/2

8. How many pounds of explosives will you need to cut four 40" diameter trees?
Use the algebraic expression: $P = \frac{D^2}{250}$

P = Pounds of explosives
D = Diameter of tree in inches
250 = constant

a. 6.4 pounds
b. 4.6 pounds
c. 26.5 pounds
d. 25.6 pounds

9. The objective is located 16 1/4 miles (distance) away, and you are moving at a <u>rate</u> of 5 mph. How long will it take you to arrive at the objective?
Use the algebraic expression: $T = \frac{D}{R}$

T = Time
D = Distance
R = Rate

a. 3.25 hours
b. 3.52 hours
c. 2.35 hours
d. 2.53 hours

10. You are in a defensive position with a front of 180 meters. Your platoon is responsible for the tactical wire emplacement of three belts. What is the total length of tactical wire (TAC(W)) required?
Use the algebraic expression:

TAC(W) = 1.25 x length of front(LOF) x number of belts (# Belts).

a. 675 meters
b. 657 meters
c. 252 meters
d. 225 meters

Complete items 11 through 13 by determining if the equations are true or false.

11. 3 + 7 = 5 x 2

a. 10 = 10 (true)
b. 10 = 7 (false)
c. 21 = 10 (true)
d. 21 = 7 (false)

12. 4 x 6 1/2 = 6 x 4 1/2

a. 27 = 26 (false)
b. 26 = 27 (false)
c. 26.5 = 26.5 (true)
d. 24.5 = 24.5 (true)

13. $7^2 - 8 \cdot 5 + 11 = 4^3 - 8^2 + 5^2$

a. 41 = 6 (false)
b. 6 = 41 (false)
c. 25 = 20 (true)
d. 20 = 25 (false)

Complete items 14 through 16 by selecting the correct root.

14. 6x - 3 = 27

a. x = 7
b. x = 6
c. x = 5
d. x = 4

15. 7h + 7.79 = 29

a. h = 4.30
b. h = 4.03
c. h = 3.30
d. h = 3.03

16. 8b + 2b + 7b + 187 = 221

a. b = 3.5
b. b = 2.5
c. b = 3
d. b = 2

Complete items 17 through 19 by determining whether these inequalities are true or false.

17. 3 + 6 < 8 - 4

a. 9 < 4 (true)
b. 9 > 4 (true)
c. 9 < 4 (false)
d. 9 > 4 (false)

18. $4(9 - 5) - \frac{8 + 2}{5} \neq 4 + 3 \cdot 2$

   a. $10 \neq 12$ (true)
   b. $12 \neq 10$ (true)
   c. $14 \neq 10$ (true)
   d. $10 \neq 14$ (true)

19. $6 \cdot \frac{12 + 8}{5} > 2 \cdot 2$

   a. $24 > 4$ (true)
   b. $24 < 4$ (true)
   c. $48 > 8$ (true)
   d. $48 < 8$ (true)

Complete items 20 through 28 by performing the action required.

20. The combat engineers reported that the mountain road has a slope of 6 percent (6 feet of rise for every 100 feet of length). How much will the road rise in 1 mile?

   a. 528 feet per mile
   b. 52.8 feet per mile
   c. 31.68 feet per mile
   d. 316.8 feet per mile

21. You are in a defensive position and have been given 20 40mm rounds (M203 grenade launcher) for every 300 feet of front. How many rounds will you receive for every 450 feet of front?

   a. 30
   b. 29
   c. 28
   d. 27

22. A total of 135 Marines crossed the stream in 40 minutes. How many Marines will cross the stream in 60 minutes? Note: Round to nearest whole number.

   a. 204
   b. 203
   c. 202
   d. 201

23. There are 950 Marines in the battalion; 2 1/2 percent are at sickbay. How many Marines are at sickbay?
Note: Round to nearest whole number.

a. 22
b. 23
c. 24
d. 25

24. The Expeditionary Force has a total of 3248 Marines. It has just been increased by 50 percent. How many Marines make up the new Expeditionary Force?

a. 4287
b. 4672
c. 4782
d. 4872

25. Seven Marines out of 12 were wounded while on patrol. What percent was wounded?

a. 17.14%
b. 17.33%
c. 58.14%
d. 58.33%

26. If 4.5 tons is 25 percent the breaking strength of the wire rope used for retrieving the HMMWV (High Mobility Multi-Purpose Wheeled Vehicle) from the mud, what is the total (100%) breaking strength of the wire rope?

a. 18 tons
b. 19 tons
c. 20 tons
d. 21 tons

27. Sixteen of the 48 rifles inspected had unserviceable stocks. What percent of the rifles had unserviceable stocks?

a. 30.33%
b. 33.33%
c. 36.33%
d. 39.33%

28. Eighty Marines (40%) of the range detail qualified expert on the rifle range. What was the total number of Marines on the detail?

a. 500
b. 400
c. 300
d. 200

UNIT SUMMARY

In this study unit you were introduced to algebra. You identified a monomial term, like term and polynomial term in an algebraic expression. You applied the proper operations to evaluate an algebraic expression and determined if an equation or an inequality was true or false. When given equations, you used the proper operations to find the correct root and you solved equations involving proportion and percentage.

Exercise Solutions

			Reference
1.	d.	$45x^3y$	3102
2.	d.	16xy, 1xy	3103
3.	a.	$2x - x + 4z^3$	3104
4.	c.	$8r^2$ (r = 5) $8(5)^2$ 8(25) 200	3105
5.	a.	$(8y)^2$ (y = 5) $(8 \cdot 5)^2$ $(40)^2$ 1600	3105
6.	c.	$a^3 - 2a^2 + a + 4$ (a = 5) $5^3 - 2(5)^2 + 5 + 4$ 125 - 2(25) + 5 + 4 125 - 50 + 5 + 4 84	3105
7.	d.	$\frac{x^2 \text{ x } z^2}{y^2}$ (x = 5, y = 2, z = 3) $\frac{5^2 + 3^2}{2^2}$ $\frac{25 + 9}{4}$ $\frac{34}{4}$ 8 1/2	3105

8. d. $P = \frac{D^2}{250}$ 3105

$P = \frac{40^2}{250}$

$P = \frac{1600}{250}$

P = 6.4 lbs per tree multiplied by 4 trees

P = 25.6 lbs of explosives

9. a. $T = \frac{D}{R}$ 3105

$T = \frac{16\ 1/4}{5}$

$T = \frac{16.25}{5}$

T = 3.25 hours or 3 hours and 15 minutes

Remember .25 = 1/4 so 1/4 x 60 (60 minutes per hour) is 15 minutes.

10. a. Length of TAC(W) = 1.25 x LOF x # belts 3105

TAC(W) = 1.25 x 180 x 3

TAC(W) = 225 x 3

TAC(W) = 675 meters

11. a. 3 + 7 = 5 x 2 3201

10 = 10 (true)

12. b. $4 \times 6\frac{1}{2} = 6 \times 4\frac{1}{2}$ 3201

26 = 27 (false)

13. d. $7^2 - 8 \cdot 5 + 11 = 4^3 - 8^2 + 5^2$ 3201

49 - 40 + 11 = 64 - 64 + 25

9 + 11 = 0 + 25

20 = 25 (false)

14. c. 6x - 3 = 27 3202

6x - 3 + 3 = 27 + 3

6x = 30

$\frac{6x}{6} = \frac{30}{6}$

x = 5

15. d. 7h + 7.79 = 29 3202

7h + 7.79 - 7.79 = 29 - 7.79

7h = 21.21

$\frac{7h}{7} = \frac{21.21}{7}$

h = 3.03

16. d. $8b + 2b + 7b + 187 = 221$ 3202

$17b + 187 = 221$

$17b + 187 - 187 = 221 - 187$

$17b = 34$

$\frac{17b}{17} = \frac{34}{17}$

$b = 2$

17. c. $3 + 6 < 8 - 4$ 3202

$9 < 4$ (false)

18. c. $4(9 - 5) - \frac{8 + 2}{5} \neq 4 + 3 \cdot 2$ 3202

$4(4) - 2 \neq 4 + 6$

$16 - 2 \neq 10$

$14 \neq 10$ (true)

19. a. $6 \cdot \frac{12 + 8}{5} > 2 \cdot 2$ 3202

$6 \cdot 4 > 2 \cdot 2$

$24 > 4$ (true)

20. d. $\frac{6}{100} = \frac{x}{5280}$ 3302

$100x = 31680$

$\frac{100x}{100} = \frac{31680}{100}$

x = 316.8 = The road rises 316.8' in 1 mile.

21. a. $\frac{20}{300} = \frac{x}{450}$ 3302

$300x = 9000$

$\frac{300x}{300} = \frac{9000}{300}$

x = 30 = 30 rounds

22. b. $\frac{135}{40} = \frac{x}{60}$ 3302

$40x = 8100$

$\frac{40x}{40} = \frac{8100}{40}$

x = 202.50 = 203

23. c. $x = .025(950)$ 3303

$x = 23.75$

2 1/2% of 950 = 23.75 = 24 Marines

2 1/2% or 2.5% is written as .025. To convert from percent to decimal, move two places to the left.

24. d. $x = 1.5(3248)$ 3303
$x = 4872$
150% of 3248 is 4872 Marines

25. d. $7 = x\%(12)$ 3303
$7 = 12x$
$\frac{7}{12} = \frac{12x}{12}$
$58.33\% = x$

26. a. $4.50 = 25\%(x)$ 3303
$4.50 = .25x$
$\frac{4.50}{.25} = \frac{.25x}{.25}$
18 tons = x
4.50 is 25% of 18

27. b. $16 = x\%(48)$ 3303
$16 = 48x$
$\frac{16}{48} = \frac{48x}{48}$
$33.33\% = x$

28. d. $80 = 40\%(x)$ 3303
$80 = .40x$
$\frac{80}{.40} = \frac{.40x}{.40}$
$200 = x$
80 is 40% of 200

STUDY UNIT 4

UNITS OF MEASUREMENT

Introduction. Measurement came about primarily because of the need for surveying real estate and for measuring quantities in the market place. One of the early problems in measurement is one that still plagues us today. This is the choice of units of measurement. The ancient cultures used body measurements to express various lengths and widths, and a cup, bucket, rock, or piece of iron to express weights or volume. The one major drawback with these choices was standardization. Measures such as the width of the palm (the hand), the distance between the tip of the thumb and the tip of the little finger of a spread hand (the span), the distance from the tip of the elbow to the tip of the longest finger (the cubit), and the distance between the finger tips of each hand when the arms are outstretched (the fathom) were used. As you can see, the quantities would vary with the size of the merchant. We have advanced greatly since ancient cultures; we now have the means to measure much more accurately. In the United States we use both the metric and English (U.S.) systems of measurement. You will be studying both of these in this study unit, and, as a result, you will gain the knowledge and skills needed to convert and compute various units of measurement.

Lesson 1. METRIC SYSTEM

LEARNING OBJECTIVES

1. Given a series of situations involving metric linear units, use the decimal system or powers of ten to convert metric units to metric units.

2. Given arithmetical problems involving various linear units, using the decimal system, convert by computing metric units to metric units.

3. Given a series of situations involving metric capacity units, use the decimal system or powers of ten to convert metric units to metric units.

4. Given arithmetical problems involving various metric capacity units, using the decimal system, convert by computing metric units to metric units.

5. Given a series of situations involving metric weight units, use the decimal system or powers of ten to convert metric units to metric units.

6. Given arithmetical problems involving various metric weight units, using the decimal system, convert by computing metric units to metric units.

4101. General

Because of the confusion with measuring systems that prevailed throughout the world, a conference of world scientists was called in France during the latter part of the eighteenth century. Based strictly on logic and science, this group developed the metric system. The basic unit of length is the meter which was originally one ten-millionth of the distance from the equator to the north pole. Liquid measure is based on the liter which was defined as the volume of a cube with an edge dimension of one-tenth of a meter (a decimeter). The basic unit of weight is the gram which was defined as the weight of a cube of pure water at the temperature of melting ice, the cube being one-hundredth of a meter on an edge (a centimeter). Since 1799, the meter has been standardized as the length of a platinum bar sealed in a case in the Archives of State in France. The standard of weight has been changed to the kilogram (1000 grams) and is represented by a platinum weight which is also in the Archives. With these basics defined, the rest of the metric system is simply based on ratios of powers of ten. Let's first look at metric linear units.

4102. Metric Linear Units

a. Metric length measurement. The basic metric unit for length is the meter. As previously mentioned, the length of the meter was at first defined as one ten-millionth of the distance from the equator to the north pole. It was later found that there had been a slight error in determining this distance. At present, the length of the meter is a bar, called the international meter, which is made of 90 percent platinum and 10 percent iridium. It is preserved at a temperature of $0^{\circ}C$ at the International Bureau of Weights and Measures near Paris, France. Two copies of this meter are in the United States, kept at the National institute of Standards and Technology, formerly the Bureau of Standards at Washington, D.C. One of these is used as the working standard and the other one for comparison. To insure still greater accuracy, these are compared at regular intervals with the international meter.

b. Tables and terms used. In the English system of measurement there are many different terms and numbers which bear no logical relation to one another. In the metric system only a few terms and but one single number. That number is 10. Let's look at some of the most often used terms (table 4-1).

Table 4-1. Metric Linear Units

kilometer	(km)	=	1000	meters = 10^{3}
hectometer	(hm)	=	100	meters = 10^{2}
dekameter	(dkm)	=	10	meters = 10^{1}
meter	(m)	=	1	meter (basic unit)
decimeter	(dm)	=	0.1	meter = 10^{-1}
centimeter	(cm)	=	0.01	meter = 10^{-2}
millimeter	(mm)	=	0.001	meter = 10^{-3}

You should try to memorize the terms in the table. The prefixes kilo, hecto, and deka are attached to the word meter when the measurement is larger than a meter. If the measurement is smaller than a meter, the prefixes deci, centi, and milli are attached to the word meter. The expressions decimeter, dekameter, and hectometer are rarely used in actual work; the value of decimeters is usually expressed in centimeters, the values of dekameters and hectometers in meters or kilometers.

c. Operating within the metric system. The powers of ten ratios simplify working within the metric system. For example, a distance of 7 meters 8 decimeters 4 centimeters can be written decimally as 7.84 meters. This can be added to another distance such as 1 meter 2 decimeters 3 centimeters (1.23) without any conversion.

Example:

```
  7.84
+ 1.23
  9.07 meters or 9 meters 0 decimeters
       7 centimeters
```

Any of the four arithmetic operations can be done with the metric system by using the rules for decimals. Converting one unit of measure to another (which is actually either multiplication or division) is done by moving the decimal points in the same manner as was explained in study unit 2 on powers of ten. This is a process that is needed regularly for changing map distances to ground distances and vice versa. Your main concern will be with centimeters, meters, and kilometers and the changing of one to another. Let's see how this is done.

(1) To change centimeters to meters, move the decimal point two places to the left. (Note in the linear measure table that meters and centimeters are two places apart.)

Example:

125,000 centimeters = 1,250 meters.

(2) To change meters to kilometers, move the decimal point three places to the left. (Note that meters and kilometers are three places apart in the table.)

Example:
1,250 meters = 1.25 kilometers.

(3) To change centimeters to kilometers, move the decimal point five places to the left.

Example:
125,000 centimeters = 1.25 kilometers.

(4) To change kilometers to meters, move the decimal point three places to the right. (Note that when changing from large units to smaller units, it may be necessary to add zeros.)

Example:
1.25 kilometers = 1,250 meters.

(5) To change meters to centimeters, move the decimal point two places to the right.

Example:
1,250 meters = 125,000 centimeters.

(6) To change kilometers to centimeters, move the decimal point five places to the right.

Example:
1.25 kilometers = 125,000 centimeters.

Let's take another look at an example of a distance and follow it through from kilometers to centimeters.

Example:
11 kilometers = 110 hectometers*
= 1,100 dekameters*
= 11,000 meters
= 110,000 decimeters*
= 1,100,000 centimeters

* Not usually used. Inserted to show the progression.

Let's look at another example, but work in the opposite direction.

Example:

50,000 centimeters = 5000 decimeters
= 500 meters
= 50 dekameters
= 5 hectometers
= .5 kilometers

Let's look at a couple of word problems.

If the first leg of the forced march was 7 kilometers long, how many centimeters long was the first leg? If your answer was 700,000 centimeters, you're correct. Remember, to change kilometers to centimeters move the decimal point five places to the left.

Let's try a harder problem.

On a 1:25,000 map, 14.3 centimeters equals how many kilometers? Was your answer 3.575 kilometers? If not, let's see what you did wrong. One centimeter on the map equals 25,000 centimeters on the ground, so 14.3 centimeters on the map equals 25,000 x 14.3, which equals 357,500 centimeters on the ground or 3.575 kilometers. If you had trouble with these problems, review this section before moving to metric capacity units.

4103. Metric Capacity Units

a. Capacity table and terms. The basic metric unit for capacity (fluid) is the liter. The liter is determined by the measurement of a cubic decimeter. The prefixes kilo, hecto, and deka are attached to the word liter when the measurement is larger than a liter. The prefixes deci, centi, and milli are attached to the word liter if the measurement is smaller than the liter (table 4-2).

Table 4-2. Metric Capacity Units

kiloliter	(kl)	=	1000 liters	= 10^3
hectoliter	(hl)	=	100 liters	= 10^2
dekaliter	(dal)	=	10 liters	= 10
liter	(l)	=	1 liter	(basic unit)
deciliter	(dl)	=	0.1 liter	= 10^{-1}
centiliter	(cl)	=	0.01 liter	= 10^{-2}
milliliter	(ml)	=	0.001 liter	= 10^{-3}

The expressions or terms kiloliter, dekaliter, deciliter, and centiliter are rarely used in actual day-to-day work. The value of kiloliter is usually expressed in hectoliters, the value of dekaliters in liters, the values of deciliters and centiliters in liters or milliliters. Our main concern will be with hectoliters, liters, and milliliters and the changing of one to another.

b. Operating within the metric capacity units. The same operations used for metric linear units also apply when working with metric capacity units. Any of the four arithmetic operations can be done using the rules for decimals or the powers-of-ten ratios. Converting one unit to another unit is done by moving the decimal points. Let's see how this is done.

(1) To change liters to milliliters, move the decimal point three places to the right. (Note in the capacity table that liters and milliliters are three places apart.)

Example:
6 liters = 6,000 milliliters.

(2) To change liters to hectoliters, move the decimal point two places to the left. (Note that liters and hectoliters are two places apart in the table.)

Example:
450 liters = 4.5 hectoliters.

(3) To change hectoliters to milliliters, move the decimal point five places to the right.

Example:
1.25 hectoliters = 125,000 milliliters.

(4) To change hectoliters to liters, move the decimal point two places to the right.

Example:
3.6 hectoliters = 360 liters.

(5) To change milliliters to liters, move the decimal point three places to the left.

Example:
750 milliliters = .75 liters.

(6) To change milliliters to hectoliters, move the decimal point five places to the left.

Example:

3,550 milliliters = .0355 hectoliters.

Let's take another example of capacity and follow it through from hectoliters to milliliters.

Example:

25 hectoliters = 250 dekaliters*
= 2,500 liters
= 25,000 deciliters*
= 250,000 centiliters*
= 2,500,000 milliliters

* Not usually used. Inserted to show the progression.

Let's try a word problem.

If the fuel tank holds 755 liters of fuel, how many hectoliters of fuel does it hold? If your answer was 7.55 hectoliters, you're correct. Remember, to change liters to hectoliters move the decimal point two places to the left.

Let's try one more.

An M1 tank, D7G bulldozer, and a 5 ton truck needed to have their fuel tanks topped off. The fuel bladder has 1892 liters of fuel remaining. The M1 tank needed 7 hectoliters of fuel, the D7G bulldozer needed 500,000 milliliters of fuel, and the 5 ton truck needed 300 liters of fuel. How many liters of fuel remain in the fuel bladder? If your answer was 392 liters, you're correct! If it wasn't, let's see what you did wrong.

The M1 tank used 7 hectoliters or 700 liters, the D7G bulldozer used 500,000 milliliters or 500 liters, and the 5 ton truck used 300 liters. Thus, 700 + 500 + 300 equals 1,500 liters. There was 1,892 liters of fuel in the bladder. Subtract 1,500 from 1,892 and your remaining fuel is 392 liters.

Note: Keep in mind the operations with powers of ten and try to associate them with the metric prefixes: kilo, hecto, centi, and etc. Remember: to change larger units to smaller units, move the decimal point to the right. To change smaller units to larger units, move the decimal point to the left.

If you had trouble with these problems, review this section before moving to metric weight units.

4104. Metric Weight Units

a. Definition. The basic metric unit for weight is the gram. The gram is defined as the weight of a cube of pure water at the temperature of melting ice. The cube is one-hundredth of a meter on each side.

b. Weight table and terms. The prefixes discussed in paragraphs 4102 and 4103 also apply to the metric weight unit, the gram. When the measurement is larger than the gram, the prefixes kilo, hecto, and deka are attached. If the measurement is smaller than the gram, the prefixes deci, centi, and milli are attached to the unit gram.

Let's look at some of the most widely used terms.

Table 4-3. Metric Weight Units

Tonne (metric ton)	(T)	=	1,000,000 grams = 10^6
kilogram	(kg)	=	1,000 grams = 10^3
hectogram	(hg)	=	100 grams = 10^2
dekagram	(dag)	=	10 grams = 10^1
gram	(g)	=	1 gram (basic unit)
decigram	(dg)	=	0.1 grams = 10^{-1}
centigram	(cg)	=	0.01 grams = 10^{-2}
milligram	(mg)	=	0.001 grams = 10^{-3}

The term tonne (metric ton) or megagram is the expression used when weighing large or bulky amounts. For measures of weight, the terms gram, kilogram, and tonne are the only three usually employed in practice. The values of hectograms and dekagrams are normally expressed as kilograms. Decigrams, centigrams, and milligrams will usually be expressed in values of the gram. Our main concern will be with grams, kilograms, and tonnes (megagrams). Let's see how they operate.

c. Operating within the metric weight units. The powers of ten or rules for decimals also apply when working within metric weight units. Changing from one unit to another unit is as easy as moving the decimal point. If the unit is larger than the gram, move the decimal to the left. When the unit is smaller than the gram, move the decimal point to the right.

(1) To change grams to kilograms, move the decimal point three places to the left.

Example:

9,754 grams = 9.754 kilograms.

(2) To change kilograms to tonnes (megagrams), move the decimal point three places to the left.

Example:

11,300 kilograms = 11.3 tonnes.

(3) To change tonnes to kilograms, move the decimal point three places to the right.

Example:

5.75 tonnes = 5,750 kilograms.

Let's take a look at another example of a weight and follow it through from tonnes (megagrams) to grams.

Example:

.5 tonne = 500 kilograms
= 5,000 hectograms
= 50,000 dekagrams
= 500,000 grams

Here is another example working in the opposite direction.

Example:

1,550,000 grams = 155,000 dekagrams
= 15,500 hectograms
= 1,550 kilograms
= 1.55 tonnes

Now let's try a couple of word problems.

The payload of the M923 (5 ton truck) is 9,080 kilograms. How many grams will the M923 carry? If you said 9,080,000, you're correct. Remember, to change kilograms to grams, move the decimal point three places to the right.

The M923 has a payload of 8,554 kilograms and is towing a trailer with a payload of 136,200 hectograms. What is the total payload in tonnes? If your answer was 22.174 tonnes, you're correct. If not, let's see what you did wrong. The vehicle payload of 8,554 kilograms equals 8.554 tonnes. (Move the decimal point three places to the left.) The trailer payload of 136,200 hectograms equals 13.62 tonnes. (Move the decimal point four places to the left.) Add 8.554 to 13.62 for a total payload of 22.174 tonnes.

If you had a difficult time with this area, review the section before moving on.

Lesson Summary. In this lesson you gained the skills needed to convert and compute metric units (linear, capacity, and weight). You also saw how easy it is to convert metric units (linear, capacity, or weight) by simply using the powers of ten (movement of the decimal point). This (powers of ten) makes the metric system easy to use. In the next lesson you will study the U.S. system of measurement. It is actually a more difficult system to learn.

Lesson 2. U.S. SYSTEM

LEARNING OBJECTIVES

1. Given a series of situations involving U.S. linear measurements, compute using like columns to line up the units of measure.

2. Given situations with various capacity measurements, compute using like columns to line up the units of measure.

3. Given situations with various weight measurements, compute using like columns to line up the units of measure.

4201. General

All of our present day units of measure have some historical background. The backgrounds of measures of capacity and weight vary and at times can be confusing. For example, there is a variance between the U.S. gallon and the British gallon, and there are even differences throughout the U.S. The International Bureau of Weights and Measures says that within the U.S. there are eight different weights for a ton, nine different volumes for a barrel, and a different bushel for charcoal, corn, and potatoes. Out of this chaotic situation, the International Bureau of Weights and Measures has tried to make some order by standardizing measures used throughout the U.S. Let's take a look them.

4202. U.S. Linear Measure

a. Units of linear measure. Unlike the metric system where there are only a few terms and but one number to remember, the U.S. system of measuring length contains many terms and numbers. The numbers in the U.S. system bear no relation to one another and can cause a bit of confusion when trying to remember which number goes with which term. Some of the terms (table 4-4) are included just for information (the areas using the rod for measurement) while others are used often enough that they should be common knowledge or committed to memory.

Table 4-4. Linear Measure Table

12 inches (in.)	=	1 foot (ft)
36 inches	=	1 yard (yd)
3 feet	=	1 yard (yd)
16 1/2 feet	=	1 rod (rd)
5 1/2 yards	=	1 rod (rd)
63,360 inches	=	1 mile (mi)
5,280 feet	=	1 mile (mi)
1,760 yards	=	1 mile (mi)
320 rods	=	1 mile (mi)

b. Expressing and computing. You can see from the table that there are, in some cases, several ways of expressing a measurement. The yard is 3 feet or 36 inches; the mile is 5,280 feet, 1,760 yards, or 63,360 inches. Usually though, measurements do not come out evenly and must be expressed in fractional parts or in terms of a smaller unit. For example, 132 inches is 11 feet or 3 2/3 yards. Conversely, 3 2/3 yards can be expressed as 11 feet or 132 inches. The form of expressing any measurement is usually determined by the preference of the user except in cases where something must be ordered in standard packages or sizes or when a standard format is used. For example, when ordering a 2 x 4 (board) that is 8 feet long, you would not ask for one 2 2/3 yards long, nor would you give them the measurement in inches.

Computations with these units can be simplified if you relate them to our decimal system. You know that when you add a column of numbers you add like columns: units, tens, hundreds, etc. This in fact is what you do when you add feet and inches.

Example:

The number (26) is also expressed as 2 tens and 6 units. Thirty-one inches is the same as 2 feet and 7 inches. Additionally 6000 feet can be expressed as 1 mile 240 yards or 1 mile 720 feet.

To compute with the various measures, then, it is necessary only to line up the proper units.

<u>Example</u>:

What is the total length of two boards if one is 3 feet 7 inches and the other is 4 feet 8 inches?

```
3 ft  7 in.
4 ft  8 in.
-----------
7 ft 15 in.
```

Since 15 inches is greater than 1 foot (12 inches), the foot is added (or carried) to the foot column and the remainder of the 15 inches is expressed in inches.

<u>Example</u>:

7 feet 15 inches = 8 feet 3 inches

As you can see, by using like columns to line up the units, the computation becomes much easier. Let's try another problem.

<u>Example</u>:

How many miles were run if the first squad ran 2,520 yards, the second squad ran 2,760 yards, and the third squad ran 2,640 yards? Let's look at two different methods to solve this.

Method 1:

```
2,520 yd
2,760 yd
2,640 yd
--------
7,920 yd
```

7,920 yards = 4 miles 880 yards or 4 1/2 miles.

Method 2:

```
2,520 yd = 1 mi 760 yd
2,760 yd = 1 mi 1,000 yd
2,640 yd = 1 mi 880 yd
----------------------
           3 mi 2,640 yd
```

3 miles 2,640 yards = 4 miles 880 yards = 4 1/2 miles

Although it is not shown, in both methods division has been used. For example, to change the 7,920 yards to miles, you divided 7,920 yards by 1,760. In method 2, you divided each distance by 1,760 and then added like columns.

In most cases it is best to change everything to the same unit. When multiplication is necessary to solve a problem, the various units can be changed (carried) as you go along, or they can be changed (carried) at the end.

Example:

You are preparing supplies for the upcoming patrol. Your squad leader has told you that there are 8 streams you must cross. All of them are 2 yards 2 feet 2 inches wide. You will cross the streams with the aid of a 3/4" fiber rope. The rope can be used only once. In other words, you will need 8 lengths of rope. What is the total length of the 8 pieces of rope? Note: Change (carry) as you multiply.

8 x 2 in. = 16 in. = 1 ft 4 in. (carry 1 ft)
8 x 2 ft = 16 ft + 1 ft = 17 ft = 5 yd 2 ft (carry 5 yd)
8 x 2 yd = 16 yd + 5 yd = 21 yd

The total rope length is 21 yards 2 feet 4 inches.

If you had trouble with either of these problems, review this section before moving on to capacity measurements.

4203. U.S. Capacity Measure

a. Units of capacity measure. The capacity measure contains many terms and numbers. The term capacity measurement refers to many types of measurement such as liquid measure and dry measure. In this section you will be concerned with liquid measure. The terms in table 4-5 are common knowledge liquid measure.

Table 4-5. Capacity Measure Table

8 fluid ounces (oz)	= 1 cup
16 fluid ounces	= 1 pint (pt)
2 cups	= 1 pint
32 fluid ounces	= 1 quart (qt)
2 pints	= 1 quart
4 quarts	= 1 gallon (gal)
128 fluid ounces	= 1 gallon
31 1/2 gallons	= 1 barrel

b. Expressing and computing. Capacity measurements can be expressed in many different ways. For example, the gallon is 4 quarts or 128 fluid ounces, and the quart is 32 fluid ounces or 2 pints. As previously mentioned, measurements do not always come out evenly and must be expressed in fractional parts or in terms of a smaller unit. Computations with liquid capacity measurements are made in the same manner as with linear measurements. Like units are placed in their proper columns.

Example:

There are two containers filled with diesel fuel. One has a capacity of 2 gallons 3 quarts and the other has 3 gallons 2 quarts. What is the total capacity of both containers?

```
2 gal 3 qt
3 gal 2 qt
5 gal 5 qt = 6 gal 1 qt
```

Note: Since 5 quarts is greater than 1 gallon, the gallon is added (or carried) to the gallon column and the remainder of the 5 quarts is expressed in quarts. Consequently, the total capacity is 6 gallons 1 quart.

Let's take a look at another example problem and two different methods of solving it.

Example:

How many gallons of water were consumed if the first squad drank 20 pints, the second squad drank 18 pints, and the third squad drank 27 pints?

Method 1:

```
20 pt
18 pt
27 pt
65 pt

65 pt = 32 qt 1 pt
32 qt = 8 gal
65 pt = 8 gal 1pt or 8 1/8 gal
```

Method 2:

```
20 pt = 2 gal 2 qt
18 pt = 2 gal 1 qt
27 pt = 3 gal 1 qt 1 pt
          7 gal 4 qt 1 pt = 8 gal 1 pt
```

Note: The total consumed was 8 1/8 gallons (method 1) or 8 gallons 1 pint (method 2). Although it was not indicated, division was used in both methods. There are many problems that you can solve by using just the division step; however, other problems will require both multiplication and division. Let's take a look at one.

Example:

Each Marine in the patrol drank 3 gallons 2 quarts 1 pint of water during training. There are 8 Marines in the patrol; how much water did the patrol drink? Note: Change (carry) as you multiply.

8 Marines x 3 gal 2 qt 1 pt

8 x 1 pt = 8 pt = 4 qt
8 x 2 qt = 16 qt + 4 qt = 20 qt = 5 gal
8 x 3 gal = 24 gal + 5 gal = 29 gal

The patrol drank 29 gallons of water.

Note: In this example you first multiplied and then divided to obtain the correct portion of pints, quarts, and then gallons.

If you had a difficult time solving either problem, review this section before moving on to weight measurement.

4204. U.S. Weight Measure

a. Units of weight measure. The weight of any object is simply the force or pull with which it is attracted toward the center of earth by gravitation. There are three common systems of weights in the U.S.: troy weight, used in weighing jewelry, apothecaries' weight, used in weighing small amounts of drugs, and avoirdupois weight, used for all ordinary purposes. Our primary concern will be with avoirdupois weight. The terms listed in table 4-6 are quite common and should be committed to memory.

Table 4-6. Weight Measure Table

* 437 1/2	grains(gr)	=	1 ounce (oz)
* 7,000	grains	=	1 pound (lb)
16	ounces	=	1 pound
2,000	pounds	=	1 ton
2,240	pounds	=	1 long ton

* A grain was originally a grain of wheat, but it is now described in relation to a cubic inch of distilled water.

b. Expressing and computing. Weight measurements can be expressed in several different ways. The ton is 2,000 pounds, the pound is 16 ounces or 7,000 grains. Here, as with linear and capacity measurements, the measurements do not always come out evenly and must be expressed in fractional parts or in terms of a smaller unit. For example, 2,500 pounds is 1 1/4 tons or 1 ton 500 pounds.

Computations with the weight measurements are made in the same manner as with linear or capacity measurements. To add or subtract, the numbers must be lined up in like columns.

Example:

Cpl Barbell has two boxes that need to be mailed to Sgt Steel. One of the boxes weighs 10 pounds 14 ounces and the other weighs 19 pounds 6 ounces. The Postmaster said, "You'll save money if the combined weight is less than 35 pounds and you mail just one box." Can Cpl Barbell save any money?

10 lb 14 oz
19 lb 6 oz
29 lb 20 oz = 30 lb 4 oz (Yes, Cpl Barbell can save money because 30 pounds 4 ounces is less than 35 pounds.

Note: Since 20 ounces is greater than 1 pound, the pound is added to the pound column and the remainder of the 20 ounces is expressed in ounces. Let's take a look at two more examples.

Example:

You have 6 pounds 14 ounces of explosives. You need charges each weighing 1 pound 6 ounces for steel cutting. How many 1 pound 6 ounce charges can you make?

$$1 \text{ lb } 6 \text{ oz} \overline{\big)\, 6 \text{ lb } 14 \text{ oz}} = \quad 22 \text{ oz} \overset{5}{\overline{\big)\, 110 \text{ oz}}}$$

$$\qquad\qquad\qquad\qquad\qquad\qquad 110 \text{ oz}$$

You can make five 1 pound 6 ounce charges.

Example:

There are 5 truck loads of supplies. Each truck load weighs 1 ton 500 lb 10 oz. What is the total weight of supplies.

5 x 10 oz = 50 oz = 3 lb 2 oz (carry 3 lb)

5 x 500 lb = 2,500 lb + 3 lb = 2,503 lb
= 1 ton 503 lb (carry 1 ton)

5 x 1 ton = 5 tons + 1 ton = 6 tons

The total weight for all supplies is 6 tons 503 pounds 2 ounces.

Note: As you can see by each example, division and multiplication are both used in finding the answers. It doesn't matter if you change the various units as you go or at the end.

Lesson Summary. This lesson taught you how to compute U.S. linear, capacity, and weight measurements. You learned that the units of linear measure are inches, feet, yards, rods, and miles, and units of capacity measure are ounces, cups, pints, quarts, gallons, and barrels. In addition, you learned that the units of weight measure are grains, ounces, pounds, and tons. In your next lesson you are going to learn how to convert U.S. and metric units.

Lesson 3. CONVERSION BETWEEN U.S. AND METRIC UNITS

LEARNING OBJECTIVES

1. Given linear measurements in metric units, convert to U.S. units.

2. Given linear measurements in U.S. units, convert to metric units.

3. Given capacity measurements in metric units, convert to U.S. units.

4. Given capacity measurements in U.S. units, convert to metric units.

5. Given weight measurements in metric units, convert to U.S. units.

6. Given weight measurements in U.S. units, convert to metric units.

4301. Conversion of Linear Measure

a. General. Hopefully, the time will come when you will no longer need to change from the metric system to the U.S. system or vice versa. However, until that time arrives, you need to know how to convert meters to yards, miles to kilometers, inches to centimeters, etc. The most important conversions for Marines involve linear measurements such as centimeters, meters, and kilometers and inches, feet, yards, and miles. Just what is the relationship between a mile and a kilometer, a yard and a meter, or a centimeter and an inch? The relationships are not quite as clear cut as working within the metric system, but with the memorization of a few conversion factors, changing meters to miles or kilometers to yards becomes a simple multiplication or division problem.

b. Converting metric linear units to U.S. linear units. As previously mentioned, the memorization of a few conversion factors help when changing metric units to U.S. units. Table 4-7 gives various conversions that will be very useful to you. If you memorize them, you will be able to calculate and convert any metric distance to the equivalent U.S. distance. Look at table 4-7 and memorize the factors. Following the table, several examples will be presented illustrating the conversion of the more important units.

Table 4-7. Conversions from Metric Units

When you know	Divide by	To Find
centimeters	2.5	inches
centimeters	30	feet
meters	0.9	yards
kilometers	1.6	miles

Note: All conversion factors are approximations; however, they are accurate enough for use in this course.

Example:
The enemy target is 1500 meters from you. How many yards away is the target?

Note: Since a meter is longer than a yard, you know that the answer will be larger than 1500. It is only necessary to remember one factor. If you know that 1 yard = 0.9 meters, the problem can be solved by division.

```
                               1666.66
0.9 | 1500      =    9 | 15000.00
```

The enemy target is 1666.66 yards away.

Example:

The convoy is moving 300 kilometers north. The first leg of the journey is 100 kilometers. How many miles will the convoy travel on the first leg? From the table you can see that 1 mile = 1.6 kilometers. This problem is easily solved by division.

```
                                62.5
1.6 | 100.00     =    16 | 1000.0
```

The convoy will travel 62.5 miles on the first leg.

Example:

The wire obstacle is 185 centimeters high. What is the height of the obstacle in feet and inches? From the table, you know that 1 foot = 30 centimeters and 1 inch = 2.5 centimeters. The problem can be solved by division.

```
       6                                     2
30 | 185.         2.5 | 5.0    =   25 | 50.
     180
       5
```

The obstacles's height is 6 feet 2 inches.

Note: The remaining 5 centimeters was divided by 2.5 to obtain inches.

You just learned how to convert metric units to U.S. units. Now let's learn how to convert from U.S. units to metric units.

c. Converting U.S. linear units to metric linear units. The conversion of U.S. units to metric units is a simple multiplication problem using the same factors that were used to change metric units to U.S. units. Try to remember the conversion factors that apply to the different units. Several examples will be presented to illustrate the conversion of the more important units.

Table 4-8. Conversions from U.S. Units

When you know	Multiply by	To find
inches	2.5	centimeters
feet	30	centimeters
yards	0.9	meters
miles	1.6	kilometers

<u>Example</u>:

The bridge to be destroyed is located 5 miles west of the city. How many <u>kilometers</u> west of the city is the bridge? From the chart you should remember that 1 mile = 1.6 kilometers; therefore, 5 miles is 1.6 times 5.

$$\begin{array}{r} 1.6 \\ \underline{\times \quad 5} \\ 8.0 \end{array} \text{ km} = 5 \text{ mi}$$

The bridge is located 8 kilometers west of the city.

<u>Example</u>:

Hill 7500 on the map is indicated as being 250 feet above sea level. How many centimeters above sea level is the hill? From the chart you know that 1 foot = 30 centimeters; therefore, 250 feet would be 30 times 250.

$$\begin{array}{r} 250 \\ \underline{\times\ 30} \\ 7500 \end{array} \text{ cm}$$

The hill is 7500 centimeters above sea level.

<u>Example</u>:

Your mission requires you to cross a river that is 45 yards wide. How many <u>meters</u> wide is the river? To obtain meters, multiply 45 yards x .9.

$$\begin{array}{r} 45 \\ \underline{\times\ .9} \\ 40.5 \end{array} \text{ m}$$

The river is 40.5 meters wide.

Example:

The vehicles were protected (hardened) with a layer of sandbags 17 inches high. What is the height of the sandbags in centimeters?

$$\begin{array}{r} 17 \\ \underline{\times\ 2.5} \\ 42.5 \text{ cm} \end{array}$$

The sandbags are 42.5 centimeters high.

Up to this point you learned how to convert linear units. Remember, to convert metric linear units to U.S. linear units, divide by the conversion factor, and to change U.S. linear units to metric linear units, multiply by the conversion factor. If you had a difficult time with these conversions, review this section before moving to conversion of capacity measure.

4302. Conversion of Capacity Measure

a. Converting metric capacity units to U.S. capacity units. Conversion between metric capacity and U.S. capacity is done the same way as with linear measure. The only difference is the conversion factors. Changing from metric units to U.S. units is just a simple division problem. Look at table 4-9 and memorize the conversion factors that apply to the different units. Following the table are several examples.

Table 4-9. Conversions from Metric Units

When You Know	Divide by	To Find
milliliters	30	fluid ounces
liters	0.47	pints
liters	0.95	quarts
liters	3.8	gallons

Example:

The motor transport operator added 150 milliliters of brake fluid to the master cylinder. How many fluid ounces of brake fluid did he add? To convert milliliters into fluid ounces, divide by 30.

$$30 \overline{)\,150\,} \quad = 5$$

The operator added 5 fluid ounces of brake fluid.

<u>Example</u>:

The guard detail drank 35 liters of milk during chow. How many pints did the guard detail drink? From the chart you see that 1 pint = 0.47 liters. The problem can be solved by dividing 35 by 0.47

$$0.47 \overline{)\,35\,} = 47 \overset{75.46}{\overline{)\,3500.\,}}$$

The detail drank 75.46 pints of milk.

<u>Example</u>:

After rebuilding the transmission on the M929 dump truck, the mechanic added 17 liters of transmission fluid. How many quarts of transmission fluid did he add?

$$0.95 \overline{)\,17\,} = 95 \overset{17.89}{\overline{)\,1700.\,}}$$

The mechanic added 17.89 quarts of transmission fluid.

<u>Example</u>:

During the first day of training, the platoon drank 83.6 liters of water. How many gallons of water did the platoon drink?

$$3.8 \overline{)\,83.6\,} = 38 \overset{22.}{\overline{)\,836.\,}}$$

The platoon drank 22 gallons of water.

You can see by these examples that conversion from metric capacity units to U.S. capacity units is accomplished through division. If you had difficulty with these conversions, review this section before moving forward.

b. Converting U.S. capacity units to metric capacity units. To change U.S. capacity units to metric units, simple multiplication is used. The approximate conversion factors are the same as the ones used to convert metric capacity to U.S. capacity. You should memorize the conversion factors to apply them to their respective units. Several examples follow the conversion table.

Table 4-10. Conversions from U.S. Units

When you know	Multiply by	To find
fluid ounces	30	milliliters
pints	0.47	liters
quarts	0.95	liters
gallons	3.8	liters

Example:

The squad used 50 fluid ounces of CLP (cleaner lubricant preservative) to clean their weapons. How many milliliters of CLP did the squad use? To convert 50 fluid ounces to milliliters, multiply 50 by 30.

$$\begin{array}{r} 50 \\ \underline{\times\ 30} \\ 1500 \end{array}$$

The squad used 1500 milliliters of CLP.

Example:

The engineers' chainsaw requires 3 pints of two-cycle oil for every three gallons of gasoline. How many liters of two-cycle oil are required? From the table you see that 1 pint = 0.47 liters.

$$\begin{array}{r} 0.47 \\ \underline{\times\quad 3} \\ 1.41 \end{array}$$

The chainsaw requires 1.41 liters of two-cycle oil for every three gallons of gasoline.

Example:

Each Marine carried 6 quarts of water while in the desert. How many liters of water did each Marine carry? Multiply 6 by 0.95 to obtain liters. 1 quart = 0.95 liters.

$$\begin{array}{r} 0.95 \\ \times \quad 6 \\ \hline 5.70 \end{array}$$

Each Marine carried 5.7 liters of water while in the desert.

Example:

The fireteam had one 5 gallon water jug. How many liters of water did they have? From the table, 1 gallon = 3.8 liters.

$$\begin{array}{r} 3.8 \\ \times \quad 5 \\ \hline 19.0 \end{array}$$

The fireteam had 19 liters of water.

Remember, converting U.S. capacity units to metric capacity units is just the opposite of converting metric capacity units to U.S. capacity units. If you had a difficult time with any of these conversions, review this section before moving to conversion of weight measure.

4303. Conversion of Weight Measure

a. Converting metric weight to U.S. weight. Converting metric weight to U.S. weight is done the same way that was covered in the two preceding lessons. The conversion is done using division and some very important conversion factors. (The conversion factors in the table are approximations and should not be used for accurate weight conversions.) Examples follow the factor conversion table.

Table 4-11. Conversions from Metric Units

When you know	Divide by	To find
grams	28	ounces
kilograms	0.45	pounds
tonne	0.9	short ton (2000 lb.)

Example:

The material for the bunker weighed 16 tonnes. How many tons does the material weigh? 1 short ton = .09 tonne.

$$0.9\,\overline{)\,16} \quad = \quad 9\,\overset{17.77}{\overline{)\,160.00}}$$

The material for the bunker weighs 17.77 short tons.

Example:

It requires 99 kilograms of explosives to breach the obstacle. How many pounds of explosives are required? 1 pound = 0.45 kilograms.

$$0.45\,\overline{)\,99} \quad = \quad 45\,\overset{220}{\overline{)\,9900}}$$

Two hundred twenty pounds of explosives are required to breach the obstacle.

Example:

The compass weighs 167 grams. How many ounces does the compass weigh? 1 ounce = 28 grams.

$$28\,\overline{)\,167} \quad = \quad 28\,\overset{5.96}{\overline{)\,167.00}}$$

The compass weighs 5.96 ounces.

As shown by the examples, conversion from metric weight to U.S. weight is done by division. If you had a difficult time with these conversions, review this section before moving forward.

b. Converting U.S. weight to metric weight. To convert U.S. weight to metric weight, you multiply the U.S. weight by the conversion factor (Table 4-12). Keep in mind that the conversions are only approximations.

Table 4-12. Conversion from U.S. Units

When you know	Multiply by	To find
ounces	28	grams
pounds	0.45	kilograms
short tons (2000 lb.)	0.9	tonnes

Example:

The mail clerk said the MCI course weighs 12 ounces. How many grams does the MCI course weigh? 1 ounce = 28 grams.

$$\begin{array}{r} 12 \\ \underline{\times\ 28} \\ 336 \end{array}$$

The MCI course weighs 336 grams.

Example:

The Marine weighed in at 160 pounds. How many kilograms does the Marine weigh? 1 pound = 0.45 kilograms.

$$\begin{array}{r} 160 \\ \underline{\times\ 0.45} \\ 72 \end{array}$$

The Marine weighs 72 kilograms.

Example:

The tank weighs 60 short tons. How many tonnes does the tank weigh? 1 short ton = 0.9 tonnes.

$$\begin{array}{r} 60 \\ \underline{\times\ 0.9} \\ 54 \end{array}$$

The tank weighs 54 tonnes.

Note: It would be a good idea to commit to memory the conversion factors and the units to which they apply.

Lesson Summary. In this lesson you converted metric linear, capacity, and weight units to U.S. units and vice versa. Remember, to change from metric to U.S. units, divide by the conversion factor and to change U.S. units to metric units, multiply by the conversion factor. Now it's time to test your knowledge. Attack the unit exercise!

Unit Exercise: Complete items 1 through 51 by performing the action required. Check your responses against those listed at the end of this study unit.

1. The first leg on the land navigation course was 43,000 centimeters long. How many meters long was the first leg?

   a. 4,300 c. 43.0
   b. 430 d. 4.30

2. The enemy is located 15 kilometers west of our position. How many meters away is the enemy?

   a. 1.5 c. 1,500
   b. 150 d. 15,000

3. The explosion was seen 600 meters north of the OP (observation post). How many kilometers away was the explosion seen?

   a. 60 c. .6
   b. 6.0 d. .06

4. The first leg of the forced march was 7 kilometers long. How many centimeters long was the first leg?

   a. 700,000 c. 7,000
   b. 70,000 d. 700

5. On a map with a scale of 1:50,000, the distance between point A and B is 18 centimeters. What is the actual ground distance in meters?

   a. 900 c. 90,000
   b. 9,000 d. 900,000

6. On a 1:25,000 map, 14.3 centimeters equals how many kilometers on the ground?

   a. 3.757 c. 37.57
   b. 3.577 d. 35.77

7. The platoon drank 155 liters of water. How many milliliters of water did the platoon drink?

   a. 1,550 c. 155,000
   b. 15,500 d. 1,550,000

8. The fuel bladder has 755 liters of diesel fuel remaining. How many hectoliters of diesel fuel remain?

a. 75.5
b. 7.55
c. .755
d. .0755

9. It takes 4,834 milliliters of fuel to prime the diesel engine. How many liters of fuel are needed to prime the engine?

a. 48,340
b. 483.4
c. 48.34
d. 4.834

10. Of 6.5 hectoliters of fuel, the private spilled 350 milliliters. How many liters of fuel did the Private spill?

a. .35
b. 3.5
c. .65
d. 6.5

11. If 750 milliliters of oil were poured from the 4.75 liter container, how many liters of oil remain?

a. 746.25
b. .74625
c. 4
d. .4

12. You have 36 liters of fuel and just added 4,000 milliliters more. How many hectoliters of fuel do you have?

a. 400
b. 40
c. 4
d. .4

13. A 3,450 kilogram pallet of ammo equals how many tonnes?

a. 34.5
b. 3.45
c. .345
d. .0345

14. The grenade weighs 500 grams. How many kilograms does the grenade weigh?

a. .5
b. 5
c. 50
d. 55

15. If the ship weighs 430 tonnes, how many kilograms does the ship weigh?

a. 4,300
b. 43,000
c. 430,000
d. 4,300,000

16. The pallet of tents weighs 750,000 grams. How many tonnes do the tents weigh?

a. 750
b. 75
c. 7.5
d. .75

17. You just removed 60,000 grams of 5.56 ammo from the stockpile of 167 kilograms. How many kilograms of ammo remain?

a. 161 c. 60
b. 107 d. 10.7

18. You have 37 tonnes plus 650,000 grams of C-4 explosives. How many kilograms of explosives do you have?

a. 37.650 c. 3,765.0
b. 376.50 d. 37,650

19. There are 4 boards measuring 3 feet 4 inches, 27 inches, 1 1/2 yards, and 2 3/4 feet. What is the total length of all 4 boards?

a. 10 ft 10 in. c. 11 ft 10 in.
b. 10 ft 22 in. d. 12 ft 10 in.

20. Each Marine in the patrol has a piece of fiber rope 5 yards 2 feet 27 inches long. There are 19 Marines in the patrol. What is the total length of all 19 segments of rope?

a. 119 yd 3 ft 6 in.
b. 120 yd 2 ft 9 in.
c. 121 yd 2 ft 9 in.
d. 122 yd 3 ft 6 in.

21. The minefield is 23 yards 1 foot wide. The engineer platoon had 11 centerline markers. The markers were placed at an equal distance apart throughout the minefield. What is the distance between each marker?

a. 6 ft 2 in. c. 1 yd 2 ft
b. 7 ft 2 in. d. 2 yd 1 ft

22. The first container had 3 gallons 3 quarts 1 pint of diesel fuel, the second container had 5 gallons 2 quarts of diesel fuel, and the third container had 6 gallons 1 quart 1 pint of diesel fuel. What is the total quantity of diesel fuel?

a. 14 gal 2 qt c. 15 gal 2 qt
b. 14 gal 3 qt d. 15 gal 3 qt

23. Each heater stove at the Mountain Warfare Training Camp burns 2 gallons 3 quarts of diesel fuel per day. There are 417 stoves in the camp. How much diesel fuel is burned daily?

a. 1,146 gal 3 qt c. 1,246 gal 3 qt
b. 1,416 gal 3 qt d. 1,426 gal 3 qt

24. You have been given 5 gallons 1 quart 2 pints of water for 11 Marines. How much water will each Marine receive?

a. 4 qt
b. 3 qt 1 pt
c. 2 qt 1 pt
d. 2 qt

25. You have 3 pallets loaded with supplies. One pallet weighs 1 ton, 600 pounds 15 ounces, another pallet weighs 6 ton 35 pounds 4 ounces, and the last pallet weighs 1,900 pounds 12 ounces. What is the total weight of the 3 pallets?

a. 8 ton 536 lb 15 oz
b. 8 ton 356 lb 15 oz
c. 7 ton 535 lb 31 oz
d. 7 ton 356 lb 31 oz

26. Each Marine has a light marching pack weighing 25 pounds 6 ounces. There were 437 Marines on the march. What is the total weight of all the packs?

a. 6 ton 925 lb 14 oz
b. 6 ton 1,088 lb 14 oz
c. 5 ton 925 lb 14 oz
d. 5 ton 1,088 lb 14 oz

27. The utilities platoon crated five reciprocating water pumps with a total weight of 490 pounds 10 ounces. What is the weight of one pump?

a. 97 lb 2 oz
b. 97 lb 4 oz
c. 98 lb 2 oz
d. 98 lb 4 oz

28. The distance that male Marines run for the PFT (Physical Fitness Test) is 3 miles. How many kilometers is this?

a. 1.6
b. 3.2
c. 4.8
d. 6.4

29. One of the combat conditioning courses is 3000 meters long. How long is the combat conditioning course in yards?

a. 31,333.33
b. 33,333.33
c. 3,133.33
d. 3,333.33

30. The tank ditch is 430 centimeters deep. How many inches deep is the ditch?

a. 162
b. 172
c. 182
d. 192

31. The wire obstacle is 430 centimeters wide. How many feet wide is the obstacle?

a. 12.33
b. 13.33
c. 14.33
d. 15.33

32. The enemy is located 65 miles east of us. How many kilometers away is the enemy?

a. 101
b. 102
c. 103
d. 104

33. Pvt. Demolition found a landmine that was 7 1/2 inches in diameter. How many centimeters in diameter is the landmine?

a. 18.75
b. 18.85
c. 19.75
d. 19.85

34. The bunker is 8 feet wide. How many centimeters wide is the bunker?

a. 200
b. 240
c. 320
d. 360

35. All Marines had to carry (firemans carry) their partners 500 yards during the evacuation drill. How many meters did they carry their partners?

a. 430
b. 440
c. 450
d. 460

36. It took 475 milliliters of boiling water to purify the canteen cup. How many fluid ounces was needed to purify the cup?

a. 18.53
b. 17.53
c. 16.83
d. 15.83

37. The Marine drank 15 liters of water a day. How many pints did the Marine drink?

a. 91.31
b. 31.91
c. 13.91
d. 11.31

38. The fuel can holds 6 liters of diesel. How many quarts does the fuel can hold?

a. 4.31
b. 5.31
c. 6.31
d. 7.31

39. Each Marine living in the barracks uses approximately 48 liters of water a day. How many gallons of water does each Marine use?

a. 11.36
b. 11.63
c. 12.36
d. 12.63

40. The engine burns 64 ounces of oil every day. How many milliliters of oil does the engine burn?

a. 1,290
b. 1,920
c. 2,019
d. 2,120

41. It took 4.5 pints of chlorine to purify the water in the holding tank. How many liters of chlorine was needed?

a. 1.215
b. 1.115
c. 2.215
d. 2.115

42. It took 8.25 quarts of paint to touch up the 5 ton truck. How many liters of paint did it take to touch up the truck?

a. 6.3785
b. 6.8375
c. 7.3785
d. 7.8375

43. The maintenance shop uses one 55 gallon drum of 30 weight oil daily. How many liters does the shop use.

a. 207
b. 208
c. 209
d. 210

44. The field jacket liner weighs 1,064 grams. How many ounces does the liner weigh?

a. 32
b. 34
c. 36
d. 38

45. The M203 grenade launcher has a total weight (with rifle) of 4.275 kilograms. How many pounds does the launcher weigh?

a. 10 lb.
b. 9.5 lb.
c. 9 lb.
d. 8.5 lb.

46. The LVS (Logistic Vehicle System) was hauling a load of supplies weighing 18 tonnes. What was the weight of the load in short tons?

a. 23
b. 22
c. 21
d. 20

47. The messhall cook used 54 ounces of butter to make pastries. How many grams of butter did the cook use?

a. 1,078	c. 1,512
b. 1,088	d. 1,688

48. The combat engineers used 1,750 pounds of C-4 to breach the minefield. How many kilograms of explosives did they use?

a. 767.5	c. 787.5
b. 777.5	d. 797.5

49. The Marines flew in 21 short tons of supplies for the refugees. How many tonnes of supplies did they fly in?

a. 19.8	c. 18.8
b. 19.9	d. 18.9

50. The MK19 40mm machinegun weighs 75.6 pounds. How many kilograms does the machinegun weigh?

a. 32.04	c. 34.02
b. 33.02	d. 35.04

51. The weight of the M9 9mm service pistol with a 15 round magazine is 1,145 grams. How many ounces does the pistol weigh?

a. 49.80 oz	c. 44.89 oz
b. 48.89 oz	d. 40.89 oz

Unit Summary

This study unit provided a brief historical account of units of measurement, and the knowledge needed to perform computations using the metric and U.S. systems. Additionally, you were taught how to perform conversions between the U.S. and metric units.

Exercise Solutions

			Reference
1.	b.	43,000 cm = 430 m	4102
2.	d.	15 km = 15,000 m	4102
3.	c.	600 m = .6 km	4102
4.	a.	7 km = 700,000 cm	4102
5.	b.	1 cm (map) = 50,000 cm (ground) 18 cm (map) = 50,000 x 18 = 900,000 cm = 9,000 m	4102

			Reference
6.	a.	1 cm (map) = 25,000 cm (ground) 14.3 cm (map) = 25,000 x 14.3 (ground) = 3,575,000 cm = 3.575 km	4102
7.	c.	155 l = 155,000 ml	4103
8.	b.	755 l = 7.55 hl	4103
9.	d.	4,834 ml = 4.834 l	4103
10.	a.	350 ml = .35 l	4103
11.	c.	4.75 l - 750 ml = 4.75 l - .75 l = 4 l	4103
12.	d.	36 l = .36 hl 4,000 ml = .04 hl 36 l + .04 l =.4 hl	4103
13.	b.	3,450 kg = 3.45 T	4104
14.	a.	500 g = .5 kg	4104
15.	c.	430 T = 430,000 kg	4104
16.	d.	750,000 g = .75 T	4104
17.	b.	167 kg - 60 kg = 107 kg	4104
18.	d.	37 T = 37,000 kg 650,000 g = 650 kg 37,000 kg + 650 kg = 37,650 kg	4104
19.	d.	3 ft 4 in. = 3 ft 4 in. 27 in. = 2 ft 3 in. 1 1/2 yd = 4 ft 6 in. + 2 3/4 ft = 2 ft 9 in. 11 ft 22 in. = 12 ft 10 in.	4202
20.	c.	19 x 5 yd 2 ft 27 in. = 19 x 27 in. = 42 ft 9 in. (carry 42 ft) 19 x 2 ft = 38 ft + 42 ft = 80 ft = 26 yd 2 ft (carry 26 yd) 19 x 5 yd = 95 yd + 26 yd = 121 yd total = 121 yd 2 ft 9 in.	4202
21.	d.	Divide 23 yd 1 ft by 10 Note: There are 10 spaces between 11 markers. 23 yd = 69 ft + 1 ft = 70 ft 70 ft ÷ 10 = 7 ft or 2 yd 1 ft	4202
22.	d.	3 gal 3 qt 1 pt 5 gal 2 qt + 6 gal 1 qt 1 pt 14 gal 6 qt 2 pt = 15 gal 3 qt	4203

		Reference
23. a.	2 gal 3 qt x 417 = 417 x 3 qt = 1,251 qt = 312 gal 3 qt 417 x 2 gal = 834 gal + 312 gal = 1146 gal 3 qt	4203
24. d.	Divide 5 gal 1 qt 2 pt by 11 5 gal 1 qt 2 pt = 44 pt 44 pt ÷ 11 = 4 pt or 2 qt	4203
25. a.	1 ton 600 lb 15 oz 6 ton 35 lb 4 oz + ,900 lb 12 oz 7 ton 2,535 lb 31 oz = 8 ton 536 lb 15 oz	4204
26. d.	437 x 6 oz = 2,622 oz = 163 lb 14 oz 437 x 25 = 10,925 lb + 163 lb = 11,088 lb = 5 tons 1,088 lb 14 oz	4204
27. c.	490 lb 10 oz = 7,850 oz 7,850 oz ÷ 5 = 1,570 oz = 98 lb 2 oz	4204
28. c.	3 mi x 1.6 = 4.8 km	4301
29. d.	3,000 m ÷ 0.9 = 3,333.33 yd	4301
30. b.	430 cm ÷ 2.5 = 172 in.	4301
31. c.	430 cm ÷ 30 = 14.33 ft	4301
32. d.	65 mi x 1.6 = 104 km	4301
33. a.	7.5 in. x 2.5 = 18.75 cm	4301
34. b.	8 ft x 30 = 240 cm	4301
35. c.	500 yd x 0.9 = 450 m	4301
36. d.	475 ml ÷ 30 = 15.83 fl oz	4302
37. b.	15 l ÷ 0.47 = 31.91 pt	4302
38. c.	6 l ÷ 0.95 = 6.31 qt	4302
39. d.	48 l ÷ 3.8 = 12.63 gal	4302
40. b.	64 fl oz x 30 = 1,920 ml	4302
41. d.	4.5 pt x 0.47 = 2,115 l	4302
42. d.	8.25 qt. x 0.95 = 7.8375 l	4302
43. c.	55 gal x 3.8 = 209 l	4302
44. d.	1,064 g ÷ 28 = 38 oz	4303
45. a.	4.275 kg ÷ 0.45 =10 lb	4303
46. d.	18 t ÷ 0.9 = 20 short ton	4303
47. c.	54 oz x 28 = 1,512 g	4303
48. c.	1,750 lb x 0.45 = 787.5 kg	4303
49. d.	21 ton x 0.9 = 18.9 t	4303
50. c.	75.6 lb x 0.45 = 34.02 kg	4303
51. d.	1,145 g ÷ 28 = 40.89 oz	4303

STUDY UNIT 5

GEOMETRIC FORMS

Introduction. Geometry is the study of the measurement and relationships of lines, angles, plane (flat) figures, and solid figures. In this study unit you will perform calculations to determine degrees, area, perimeter, volume, and circumference. Throughout your career, you will use the measurements and relationships studied here to perform a variety of tasks such as map reading and land navigation, determining fields of fire, calculating the amount of water and fuel for training operations, and determining the storage area for supplies. In fact the list really could go on. The first topic in our study is angles. Let's take a look at them.

Lesson 1. ANGLES

LEARNING OBJECTIVES

1. Given a series of angles, apply the operations for measuring angles with a protractor to obtain degrees.

2. Given a series of angles and a protractor, measure the angles to classify them.

3. Given two or more angle readings, compute into degrees, minutes, and seconds, using the operations for computation of angles.

5101. Measuring Angles

a. Definition. An angle is a set of points consisting of two rays (sides) and a common end-point (fig 5-1). Angle BAC is abbreviated BAC. It can also be written as angle CAB; they are the same angle. Point A which is the middle letter in the angle is called the vertex of the angle.

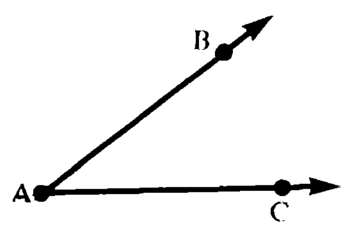


Fig 5-1. Ray AB + ray AC = angle BAC.

An angle is referred to in one of several ways (fig 5-2). It may be called angle B, angle ABC or CBA, or it may be labeled with a small letter b or numeral 2.

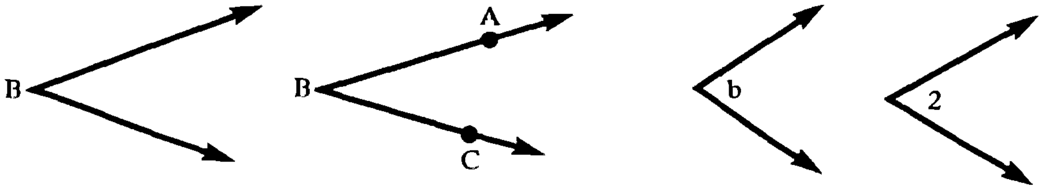


Fig 5-2. Labeling an angle.

Now that you know how to label angles, let's find out how they are measured.

b. Measurement. Although the angle is the set of points on the two rays, the measure of the angle is the amount of "opening" between the rays in the interior of the angle. To measure an angle, we must have a unit of measure and a tool inscribed with those units. The unit is the degree ($^{\circ}$) and the tool is the protractor. The protractor which comes with this course has two scales. Let's take a look at it. The inner scale measurement is in degrees and the outer scale measurement is in mils. We will only be using the inner scale. To avoid confusing the two scales, it is recommended that you cut off the mils (outer) scale.

Looking at your protractor, you will see that the degrees range from 0-360 (360 is also positioned at 0), making a complete circle. The degrees on your protractor are determined by a set of 360 rays drawn from the same center point. These rays which are numbered from 0 to 360 make 360 angles all the same size. One of these angles is the standard unit or degree for measuring angles. Figure 5-3 illustrates a scale from 0° to 180° and the actual size of 1°.

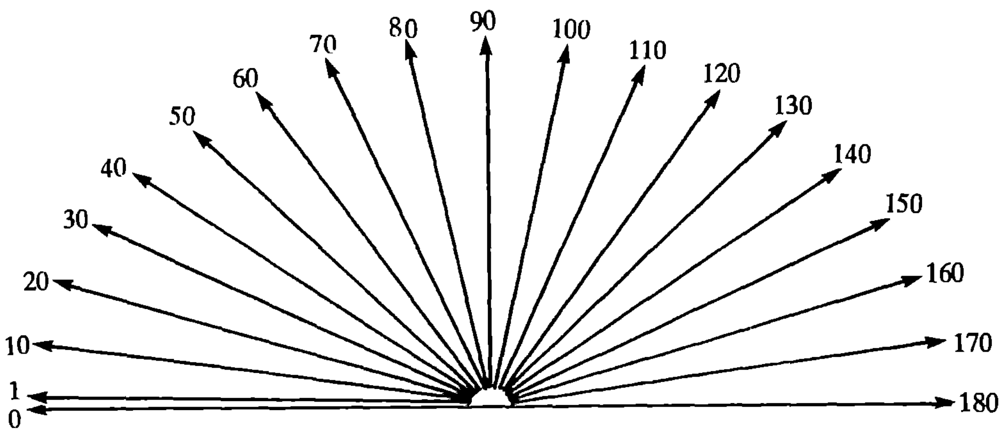


Fig 5-3. Measuring scale for angles.

Note: This protractor is normally used with the "0" pointed in the direction of north when used for land navigation. You won't be using it that way in this course. When determining the degrees of an angle, the 0/180 line becomes the base line. The base line is positioned on the angle so that you can read the scale in a clockwise direction.

To measure an angle with the protractor, place it on the angle so that the center point (crosshairs of the protractor) is on the vertex (A) of the angle and position the 0/180 line so that you read the angle measurement in a clockwise direction from zero up the scale towards 180 (fig 5-4). Observe the degree reading where the protractor scale lines up with the other ray. The number that corresponds to this ray is the measurement, in degrees, of the angle. Angle "A" in this illustration measures 35°.

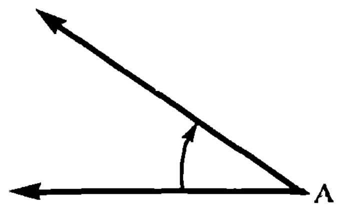
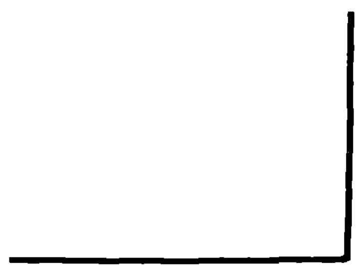


Fig 5-4. Measuring an angle.

Now use the protractor provided to measure the angles below. If the rays are not long enough, lay the edge of your protractor along the side of the angle or carefully extend the rays with a pencil.

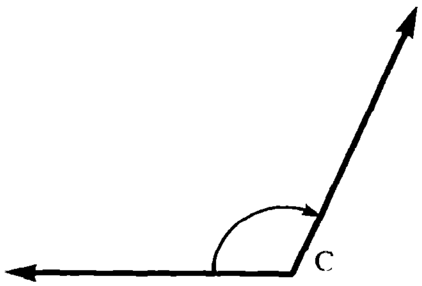
Example:



This is a 90° angle. When you measured this angle with your protractor, you should have placed it on the angle so that the center point (crosshairs of the protractor) was on the vertex of the angle. You should have positioned the 0/180 line so that you could read the angle measurement from zero in a clockwise direction up the scale towards 180. The degree reading on the protractor which lies on the other ray is 90°.

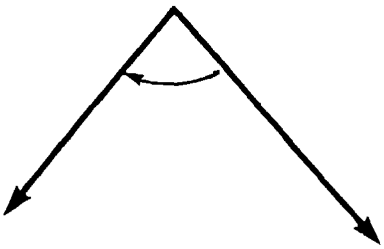
Let's try another angle. Measure angle C in the example below.

Example:



Angle "C" is a 114° angle. Remember, you must place the center point of the protractor on the vertex of the angle and position the 0/180 line to read up the scale.

Example:



Was your answer for angle "D" 79°? If not you need to go back and review the section.

5102. Angle Classification

Classifications. Angles are classified according to the number of degrees they contain (fig 5-5). An angle of 90° is called a right angle and the rays are said to be perpendicular. An angle less than 90° is called an acute angle. An angle whose measure is more than 90° and less than 180° is called an obtuse angle. An angle of exactly 180° is called a straight angle.

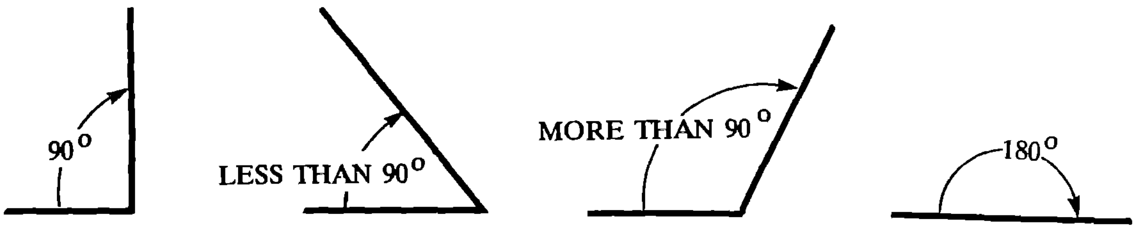


Fig 5-5. Angle classifications.

Can you look at an angle and classify it? If you can, you're ready to start computing angle readings. If not, take a few minutes to study these angles (fig 5-5). Note how they look and approximately how many degrees they measure.

5103. Computation of Angle Readings

Computations. So far, all of the angles discussed have been measured in even degrees. As a Marine, you land navigate using readings on a compass in degrees and occasionally use parts of a degree to compute yearly declination on older maps. All Marines must know how to add and subtract various angles to change compass headings, figure back azimuths, compute yearly declination, and to accurately plot on a map. Here you will learn a little more than just degrees. You will learn about minutes (') and seconds (") too! Each degree contains 60 minutes and each minute contains 60 seconds. Let's work a few problems involving minutes and seconds. Addition and subtraction of angles is done the same way as other units of measure using the decimal system. Keep the proper units in the same column and carry or borrow as needed.

Example:

$$\begin{array}{r} 12^\circ14'36'' \\ +\ 22^\circ56'42'' \\ \hline 34^\circ70'78'' \end{array} = 35^\circ11'18''$$

Note: You should see that 78" is 1' with 18" remaining. 70' + 1' is 71' which is 1° with 11' remaining. 34° plus 1° is 35°. Your answer is 35°11'18"

Example:

$$\begin{array}{r} 120^\circ45'22'' \\ +\ \ 16^\circ17'38'' \\ \hline 136^\circ62'60'' \end{array} = 137^\circ3'$$

Example:

$$\begin{array}{r} 42^\circ12'30'' \\ -\ 12^\circ30'50'' \\ \hline \end{array} = \begin{array}{r} 41^\circ71'90'' \\ -\ 12^\circ30'50'' \\ \hline 29^\circ41'40'' \end{array}$$

Note: Be careful with this problem. You cannot subtract 50" from 30" nor 30' from 12' so you must borrow 1' or 60" from 12' and add it to 30" and borrow 1° or 60' from 42° and add it to 12' before you subtract. Let's look at another example.

Example:

$$\begin{array}{rcr} 112^\circ 4'25'' & = & 111^\circ 64'25'' \\ \underline{-\ 106^\circ 5'24''} & = & \underline{-\ 106^\circ\ 5'24''} \\ & & 5^\circ 59'1'' \end{array}$$

Note: You should see from the preceding examples that, as long as you keep the proper units in the same column and borrow or carry as needed, the problem is easily solved.

Lesson Summary. During this lesson you were provided with the procedures used to measure angles, classify angles, and compute angle readings in degrees, minutes and seconds. In the next lesson you will determine dimensions of plane geometric figures.

Lesson 2. PLANE FIGURES

LEARNING OBJECTIVES

1. Given the dimensions of plane geometric figures, use the proper formula to compute the perimeter.

2. Given the dimensions of plane geometric figures, use the proper formula to compute the area.

3. Given the radius of various circles, use the proper formula to compute the circumference.

4. Given the radius of various circles, use the proper formula to compute the area.

5201. Perimeter and Area of Squares

a. General. There is an unlimited number of figures or shapes in a plane. A plane has no boundaries although we usually visualize it as looking like a piece of paper. For our purposes, we will think of a plane simply as being a flat surface with two dimensions. This piece of paper, if we forget that it actually has a thickness, represents a plane. If you took a pair of scissors and cut out various shapes from this page, each would be a plane figure. Some of these would have practical uses, others would not due to having too many sides that are too irregular in shape. The majority of the objects around you have some regular geometric shape. Let's look at some of them.

b. Square. This familiar figure has four sides of equal length and a right angle at each corner. Opposite sides are parallel. Two lines are parallel if they do not intersect, no matter how far extended. There are two important measures of a square: the perimeter and the area.

(1) Perimeter. To find the perimeter or distance around a square, add the four sides or even more simply: Perimeter (P) = 4s (4 x length of 1 side).

Example:
What is the perimeter of a square with a side that measures 16 in.?

$$P = 4s$$
$$P = 4(16)$$
$$P = 64$$

The perimeter is 64 inches.

Using the equation-solving techniques, if the perimeter is known, you can determine the length of a side. For example, if the perimeter of a square is 62 inches, what is the length of one side?

Example:

$$P = 4s$$
$$62 = 4s$$
$$\frac{62}{4} = \frac{4s}{4}$$
$$\frac{62}{4} = s$$
$$15.5 = s$$

Therefore, each side is 15 1/2 inches long.

(2) Area. Area, not only for a square, but for plane figures, is described or measured in terms of square units. A square with sides of 3 inches contains 9 squares, each 1 inch on a side (fig 5-6).

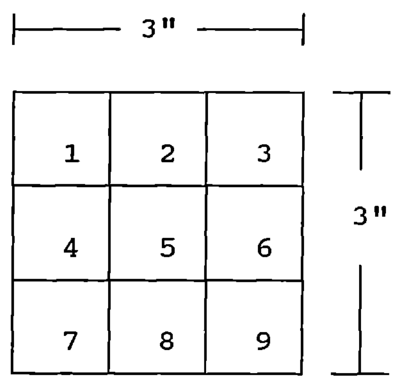


Fig 5-6. Area of a square with s = 3.

You could prove to yourself, by drawing squares of various sizes and marking them off into grids, that the area of a square is equal to the product of two of the sides or the square of one side. $A = s \times s$ or $A = s^2$.

Example:
Grenade range 303 has a square parking lot with one side that is 32 feet long. What is the area of the parking lot?

$A = s^2$
$A = 32^2$
$A = 1024$

The area is 1024 square feet.

Do you remember how to find the perimeter of a square? If you said $P = 4s$, you're right! How about the area of a square? If your answer was $A = s^2$, you're right again! If you had a difficult time understanding perimeter or area, review this section before moving on.

5202. Perimeter and Area of Rectangles

a. Rectangle. This is another familiar figure. Each pair of opposite sides are equal in length and parallel, and there is a right angle at each corner. The two important measures are perimeter and area. Let's look at them.

(1) As with the square, you can find the perimeter by adding up the measures of the four sides. The longer dimension is called the length (l) and the shorter is called the width (w). There are two lengths and two widths to a rectangle. The formula is: $P = 2l$ (2 x length) + $2w$ (2 x width).

Example:
The engineers cut a sheet of plywood 7 feet long and 4 feet wide. What is the perimeter of the sheet of plywood?

$P = 2l + 2w$
$P = 2(7) + 2(4)$
$P = 14 + 8$
$P = 22$

The perimeter is 22 feet.

Note: You can also find a missing dimension if you know the perimeter and one of the two dimensions. Let's see how to do this.

Example:
The perimeter of the momat (fiberglass matting) staging lot is 82 feet and the length is 31 feet. What is the width?

$$\begin{aligned} P &= 2l + 2w \\ 82 &= 2(31) + 2w \\ 82 &= 62 + 2w \\ 82 - 62 &= 62 - 62 + 2w \\ 20 &= 2w \\ \frac{20}{2} &= \frac{2w}{2} \\ 10 &= w \end{aligned}$$

The width is 10 feet.

Example:
In front of the barracks there is a rectangular shaped concrete slab with a perimeter of 476 feet and a width of 70 feet. What is the length of the concrete slab?

$$\begin{aligned} P &= 2l + 2w \\ 476 &= 2l + 2(70) \\ 476 &= 2l + 140 \\ 476 - 140 &= 2l + 140 - 140 \\ 336 &= 2l \\ \frac{336}{2} &= \frac{2l}{2} \\ 168 &= l \end{aligned}$$

The length is 168 feet.

(2) Area. Area is described and measured in terms of square units. As with the square, you can segment a rectangle using a grid and count the number of units (fig 5-7).

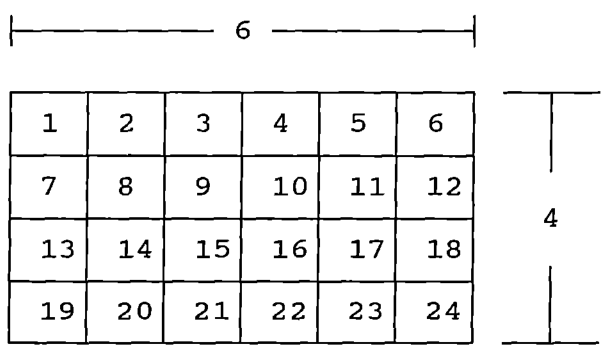


Fig 5-7. Area of a rectangle.

You can see in figure 5-7 that the number of square units is equal to the product of the two dimensions of the rectangle. This gives the formula $A = lw$.

Example:
What is the area of a rectangle that is 13.5 inches long and 6.2 inches wide?

$$A = lw$$
$$A = (13.5)(6.2)$$
$$A = 83.7$$

The area is 83.7 square inches.

Again, as you did before, you can find an unknown dimension if you know the area and one of the dimensions.

Example:
The top of the grenade box has an area of 176 square inches with a length of 16 inches. What is the width?

$$A = lw$$
$$176 = 16w$$
$$\frac{176}{16} = \frac{16w}{16}$$
$$11 = w$$

The width is 11 inches.

Example:
The area of the foundation for the bunker is 417 square feet with a width of 12 feet. How long is the foundation?

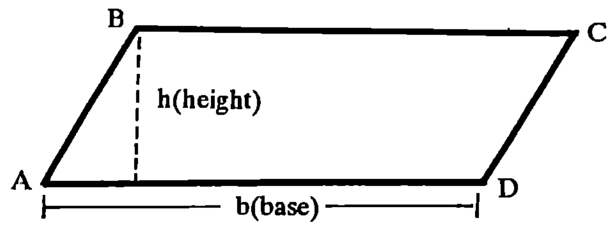
$$A = lw$$
$$417 = l(12)$$
$$\frac{417}{12} = \frac{l(12)}{12}$$
$$34.75 = l$$

The length is 34.75 feet.

Before you move on to parallelograms, do you recall the formulas used to compute the perimeter and area of a rectangle? If you said $P = 2l + 2w$ and $A = lw$ you're right! If you didn't, you should review this section before moving on.

5203. Perimeter and Area of Parallelograms

a. Parallelogram. A parallelogram is a quadrilateral (four sided figure) with each pair of sides equal and parallel. It looks like a rectangle that has been shoved sideways (fig 5-8). Opposite angles are also equal; angle A equals angle C and angle B equals angle D. The long dimension of a parallelogram is usually referred to as the base (b). The perpendicular distance between the bases is called the height (h).



AB = CD and AB//CD
BC = AD and BC//AD

Note: // means parallel

Fig 5-8. Parallelogram.

b. Perimeter. As with the other quadrilaterals, the perimeter is the sum of the lengths of the 4 sides. Two of the sides are called bases (b) and the other two sides that are slanted are called widths (w), so the formula would then be P = 2b + 2w.

Example:

What is the perimeter of a parallelogram that has a base of 22 inches and a width of 12 inches?

P = 2b + 2w
P = 2(22) + 2(12)
P = 44 + 24
P = 68

The perimeter is 68 inches.

c. Area. If you placed a grid over a parallelogram as was done with the square and rectangle, you would get fractional squares due to the sloping sides (fig 5-9). By cutting off the three fractional squares on one side and joining them with the parts of squares on the other side, you get three complete squares and a complete rectangle.

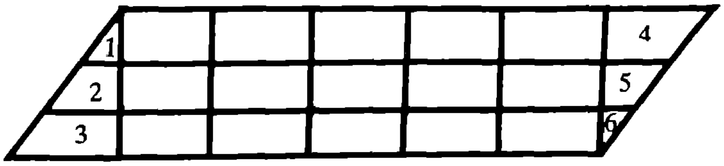
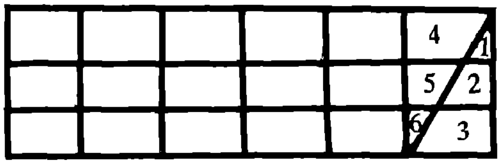


Figure 5-9. Making a parallelogram into a rectangle.

Now you can borrow the formula for the area of a rectangle and substitute the letters used for parallelograms (fig 5-10).



Area = base x height
A = bh

Fig 5-10. Formula for the area of a parallelogram.

Example:

The bottom of the enemy trenchline is shaped like a parallelogram with a base of 50.2 feet and a height of 15.3 feet. How much area does the bottom of the trenchline cover?

A = bh
A = 50.2(15.3)
A = 768.06

The area is 768.06 square feet.

Remember, to find the perimeter of a parallelogram, use the formula: P = 2b + 2w, and to find the area, use the formula: A = bh.

5204. Perimeter and Area of Trapezoids

a. Trapezoid. This is a quadrilateral with only one pair of opposite sides parallel (they are not equal). As with the parallelogram, the "top" and "bottom" of a trapezoid are referred to as bases. Since they are of different lengths, the subscripts 1 (b_1) and 2 (b_2) are used to identify them (fig 5-11).

The perpendicular distance between the parallel bases is called the height (h).

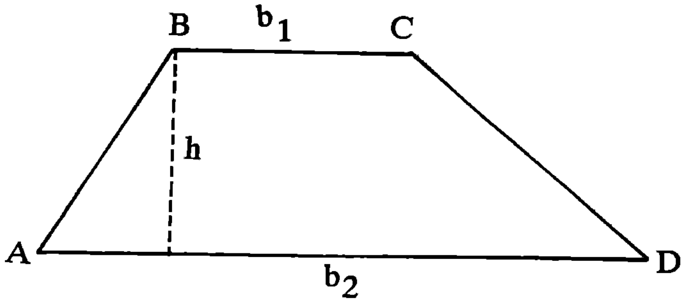


Fig 5-11. Trapezoid.

b. Perimeter. To find the perimeter, all you have to do is add the lengths of the four sides. P = a + b + c + d.

Example:

A trapezoid with side "a" 24 inches, side "b" 12 inches, side "c" 18 inches, and side "d" 12 inches has a perimeter of how many inches?

P = a + b + c + d
P = 24 + 12 + 18 + 12
P = 66

The perimeter is 66 inches.

c. Area. The formula for the area of a trapezoid is derived from the formula for the area of a triangle. A trapezoid can be divided into two triangles of unequal area (fig 5-12).

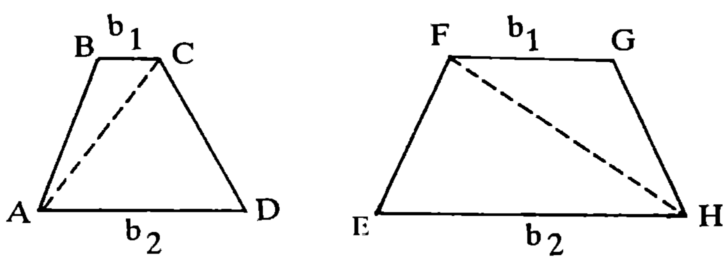


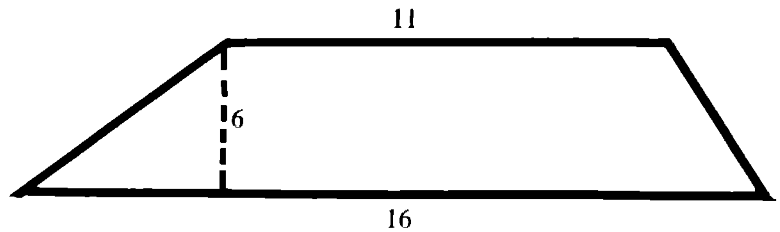
Fig 5-12. Dividing a trapezoid into two unequal triangles.

Follow along closely. To find the area of trapezoid ABCD (fig 5-12), you could add the area of triangle ABC to the area of triangle ACD. To find the area of EFGH, add triangle EFH to triangle FGH. But instead of finding two separate areas and then adding them together, a little algebraic manipulation with the formula for the area of a triangle gives us a formula for the area of a trapezoid. The triangle formula A = 1/2bh will be explained later in the study unit but will be used here without explanation.

In figure 5-12, the area (A) of ABCD is equal to $1/2b_1h + 1/2h(b_1 + b_2)$ or could be written as $A = 1/2h(b_1 + b_2)$. This is beyond the algebra that was covered in study unit 3, but if you remember how to factor, you should see how it was derived. Let's look at an example using this formula.

Example:

The enemy bunker was constructed in the shape of the trapezoid illustrated. What is the area of the trapezoid?



$A = 1/2h(b_1 + b_2)$
$A = 1/2(6)(11 + 16)$
$A = 3(27)$
$A = 81$

The area is 81 square feet.

So, to find the perimeter of a trapezoid use the formula P = a + b + c + d. Now, how do you compute the area? If your said, use the formula $A = 1/2h(b_1 + b_2)$ you're absolutely correct!

5205. Triangles. Triangle ABC is the set of points A, B, and C and all points of AB on a line between A and B, all points of AC on a line between A and C, and all points of BC on a line between B and C. The points A, B, and C are called the vertices (plural of vertex) of the triangle. A triangle may also be thought of as points A, B, and C, and the line segments that connect them. Note that in both of these descriptions, no mention is made of the inside or outside areas. The triangle is only that part represented by the line. Everything inside is the interior of the triangle; everything outside is the exterior (fig 5-13).

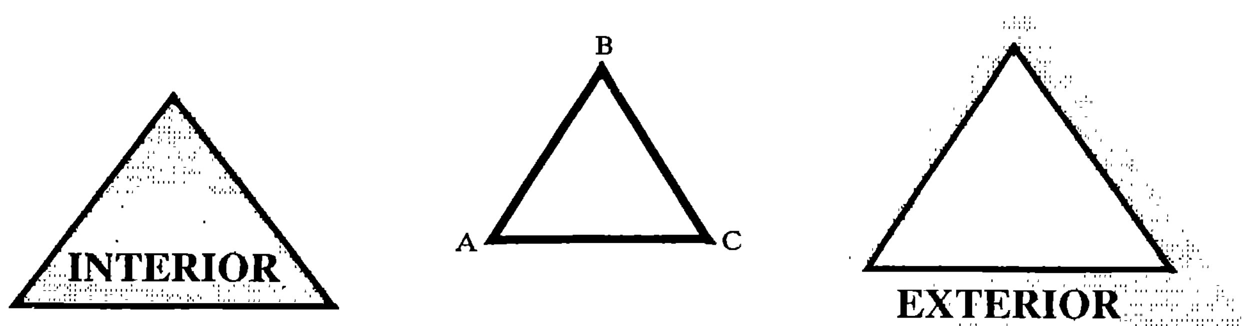
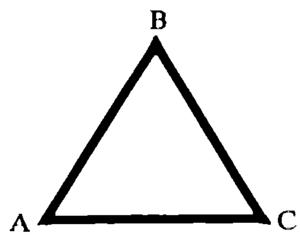


Fig 5-13. A triangle, its interior, and its exterior.

a. Types: A triangle can be classified according to the lengths of its sides or the measures of its angles. The sum of the measures of the angles of any triangle is always 180°. Let's look at the most common types of triangles.

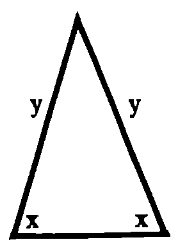
(1) Equilateral. As the name implies, all three sides are the same length. The equilateral triangle is also called equiangular (equal angles). If all of the sides (A,B,C) are the same length, then all of the angles are the same measure (60°).

Example:



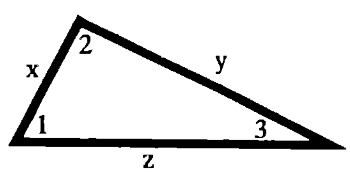
(2) Isosceles. A triangle with two equal sides (y), and consequently, two equal angles (x). The equal angles are opposite the equal sides.

Example:



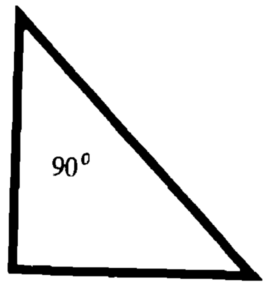
(3) Scalene. A triangle in which all three sides (x,y & z) are of different length. All three angles (1,2 & 3) are also of different measure.

Example:



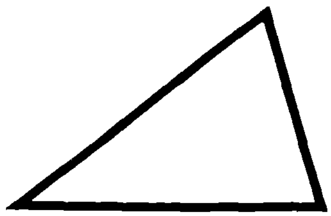
(4) <u>Right triangle</u>. A triangle in which one of the angles measures 90°.

<u>Example</u>:



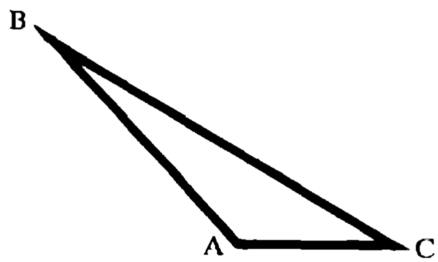
(5) <u>Acute</u>. A triangle in which all three angles have measures less than a right angle.

<u>Example</u>:



(6) <u>Obtuse</u>. A triangle in which one of the three angles has a measure greater than a right angle. In this case, angle A is greater than 90°.

<u>Example</u>:



b. <u>Perimeter</u>. The perimeter of a triangle is found by adding the lengths of the three sides: $P = a + b + c$.

<u>Example</u>:

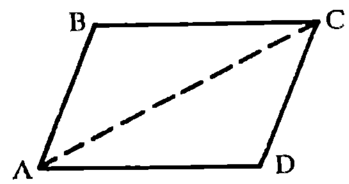
What is the perimeter of a triangular shaped field that measures 15 feet by 15 feet by 20 feet?

$P = a + b + c$
$P = 15 + 15 + 20$
$P = 50$

The perimeter is 50 feet.

c. Area. The formula for the area of a triangle can be derived from the area of a parallelogram. Any two equal triangles can be placed together to form a parallelogram. As illustrated below, the parallelogram has been divided into two equal triangles ABC and ADC.

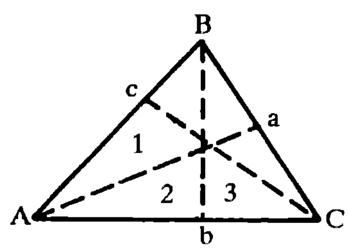
Example:



The formula used to determine the area of a parallelogram is A = bh. Since a parallelogram contains two equal triangles, the area of one of them would be half that of the parallelogram or 1/2bh.

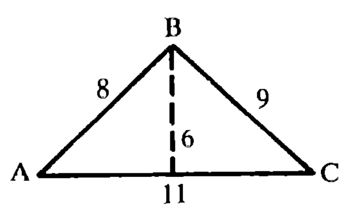
Example:

In the illustration below, if b is used as the base, 2 is the height; if c is the base, 3 is the height; and if a is the base, then 1 is the height. It doesn't matter which dimension you call the base or which the height. With real dimensions, any of the combinations would give the area of the triangle.



Example:

Find the area of the triangle illustrated. Use 11 as the base and 6 as the height.



A = 1/2bh
A = 1/2(11 x 6)
A = 33

The area of the triangle is 33 square units.

Do you remember how to find the perimeter and area of a triangle? To find the perimeter of a triangle use the formula $P = a + b + c$. To find the area of a triangle use the formula $A = 1/2bh$. Make sure you know how to use these formulas before continuing.

5206. Circles. A circle is a closed curve all of whose points are the same distance from a fixed point called the center. The center is usually labeled with a capital letter and the circle can be referred to by this letter. As with the other plane figures, a circle is only the points on the curve that represents the circle. Everything outside is the exterior; everything inside the closed curve is the interior. Let's take a look at the nomenclature of a circle (fig 5-14).

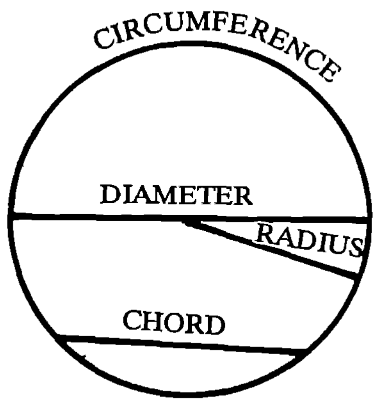


Fig 5-14. Nomenclature of a circle.

a. Nomenclature. The set of points that make up the circle is called the circumference. This is to the circle as the perimeter is to quadrilaterals. The fixed distance from the center to any point on the circle is called the radius. A line segment having its endpoints on the circle is called a chord. As figure 5-14 shows, a chord does not necessarily pass through the center. If it does, it is called the diameter of the circle. The diameter is equal to the length of two radii ($d = 2r$). An important concept about circles is that the ratio of the circumference of a circle to its diameter is a constant value 3.141592. It is symbolized by the Greek letter π (pi); this number has been known and used since ancient times. You could derive this value for yourself by measuring the circumference and the diameter of an object and then dividing the circumference by the diameter: $c/d = \pi$. Pi is usually rounded off to the decimal 3.14 or the fractions 3 1/7 or 22/7.

b. Circumference. Since circumference has been defined and its relationship to π and the diameter has been shown, it should be easy to determine the formula for computing the circumference (C) of a circle. If $c/d = \pi$, then you can manipulate this equation to find the value of c. $\cancel{d} \times \frac{c}{\cancel{d}} = \pi d$, $C = \pi d$

Since $d = 2r$, the formula could also be $C = 2\pi r$. Let's see how these two formulas are used.

Example:

Find the circumference of a water bladder (storage tank for water) that has a radius of 5 feet.

$C = 2\pi r$
$C = 2(3.14)5$
$C = 31.4$

The circumference is 31.4 feet.

Example:

Find the circumference of the 155 howitzer barrel. It has a diameter of 155mm.

$C = \pi d$
$C = 3.14(155)$
$C = 486.7$

The circumference is 486.7mm.

The following example shows how to find the radius when the circumference is known.

Example:

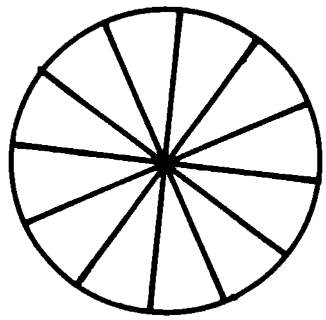
One of the trees used for the log obstacle has a circumference of 56.52 inches. What is the radius of the tree.

$$C = 2\pi r$$
$$56.52 = 2(3.14)r$$
$$56.52 = 6.28r$$
$$\frac{56.52}{6.28} = \frac{6.28r}{6.28}$$
$$9 = r$$

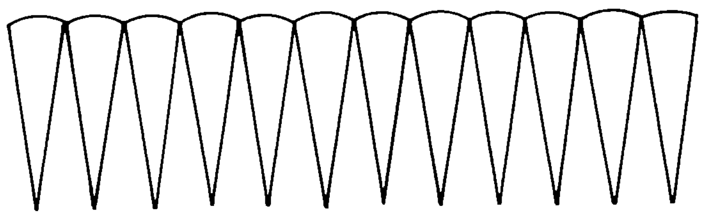
The radius of the tree is 9 inches.

Let's test your memory. If you know the radius of a circle and want to find out its circumference, what is the formula used? If your answer was $C = 2\pi r$, you're correct! How would you determine the circumference when you know its diameter? If your answer is $C = \pi d$, you're right again! If your answers were different from those given, review this section before moving on to area of circles.

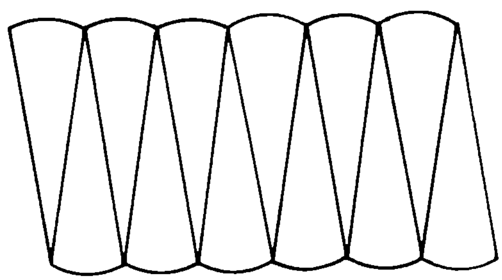
5207. Area of circles. The area of a circle is always measured in square units. It would be extremely difficult to determine the number of whole units by placing a grid over the circle as with rectangles; therefore, a formula is used. The formula was derived from the rectangle area formula, and can be described using an illustration (fig 5-15).



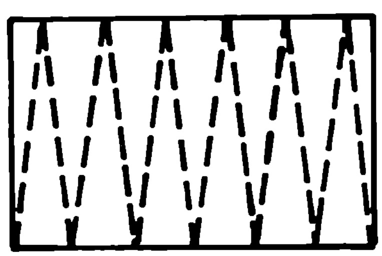
Divide a circle into any number of equal-size pie shaped sections.



Open the circle so that the sections are in a line.



Interlock the sections so that a figure approaching a rectangle is formed.



If the circle could be separated into enough sections, the smaller they got, the flatter the arcs would be. Eventually the figure would be a rectangle.

Fig 5-15. Deriving the area of a circle.

If you substitute the appropriate symbols for parts of the circle into the formula for the area of a rectangle, you will get the formula for the area of a circle. The length of the rectangle that has been formed is related to the circumference of the circle in that the top and bottom of the rectangle are the arcs that were originally the circumference (fig 5-15). Since the circumference of a circle can be expressed by the formula $2\pi r$, the length of a rectangle would be half of this or $1/2 \times 2\pi r$. The width of the rectangle is the radius of the circle. You can now substitute the variables into the formula for the area of a rectangle.

$$A = l \times w$$
$$A = \frac{1}{2} \times 2\pi r \times r = \frac{1}{\cancel{2}} \times \cancel{2}\, \pi r \times r$$
$$A = \pi r \times r$$
$$A = \pi r^2$$

<u>Example</u>:

What is the size of the casualty area for the M16A1 antipersonnel (AP) mine if the radius of the area is 30m?

$A = \pi r^2$
$A = 3.14(30^2)$
$A = 3.14(900)$
$A = 2,826$

The area is 2,826 square meters.

<u>Example</u>:

The diameter of the bull's-eye on the Able target at the rifle range is 12 inches. What is the area of the bull's-eye?

$A = \pi r^2$
$A = 3.14(6^2)$
$A = 3.14(36)$
$A = 113.04$

The area is 113.04 square inches.

<u>Note</u>: The diameter was used in this example. Remember that the radius is 1/2 the diameter.

<u>Example</u>:

During physical training (PT), the squad formed a circle with a radius of 10 feet.

What is the area of the PT circle?

$A = \pi r^2$
$A = 3.14(10^2)$
$A = 3.14(100)$
$A = 314$

The area is 314 square feet.

Lesson Summary. In this lesson you studied and applied the formulas used to determine the dimensions of plane geometric figures. In the next lesson you will apply the formulas used to determine dimensions of solid figures.

Lesson 3. SOLID FIGURES

LEARNING OBJECTIVES

1. Given the dimensions of solid geometric figures, use the proper formula to find the surface area.

2. Given the dimensions of solid geometric figures, use the proper formula to compute the volume.

3. Given a number, use the proper operations to find the square root.

4. Given the dimensions of two sides of a triangle, use the proper formula to find the dimension of the unknown side.

5. Given the dimensions of two sides of one triangle and the dimensions of one side of another triangle, use the proper operations to find the dimensions of the missing corresponding side.

5301. Surface Area and Volume of Solid Geometric Figures

a. General. A more appropriate title for this next group of geometric figures might be three dimensional figures but they are called solid geometric figures. Even though they may be hollow, there are points in this "hollow" area that help to form the solid figure. The unit of measure for a solid figure is a cube, 1-unit of length on each edge (fig 5-16). When you find the volume of a solid figure, you are finding out how many 1-unit cubes or cubic units are contained in the figure. Notice that a third dimension has been added.

In plane figures, on the other hand, you dealt with base and height or length and width. You now have length(l), width(w), and height(h) contained in one figure.

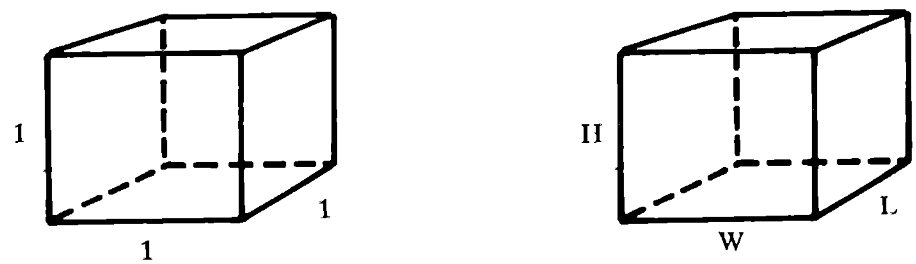
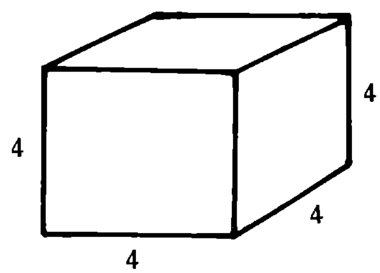


Fig 5-16. One unit cube.

b. <u>Cube</u>. The familiar cube is a geometric figure with six square <u>faces</u>. The <u>edges</u> all have the same measure. The sum of the areas of the six faces is called the <u>surface area</u>.

<u>Example</u>:

The cube illustrated represents a company supply box. There are 6 faces, each measuring 4 feet by 4 feet (4 x 4). What is the surface area (SA) for the box shown below?

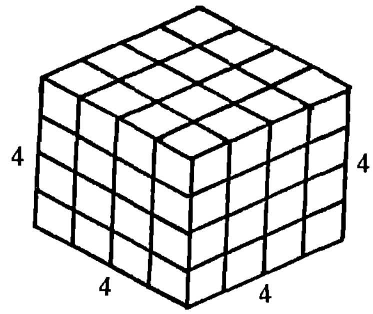


SA = $6(\text{face}^2)$
SA = $6(4^2)$
SA = 6(16)
SA = 96

The surface area equals 96 square feet.

To find the volume (V) of a cube you must find the number of <u>unit cubes</u> it contains.

Example:



If you constructed a cube as shown above and then counted the unit cubes, it would not take long to generalize the formula for the volume of a cube: V = lwh. Remember, though, that a cube has the unique characteristic of having square faces that are all the same size. The l,w, and h (the edges) are the same. Consequently, the formula can be changed to V = f x f x f or $V = f^3$.

Example:

Suppose your mission requires you to transport several supply boxes (4' x 4' x 4') using a 5 ton truck. To do this you must know how many cubic feet the 5 ton truck can hold and how many cubic feet are in each supply box.

Note: In this example, we'll figure the cubic feet for only one box.

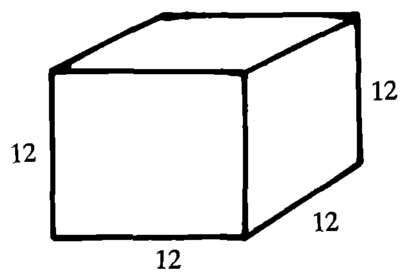
$V = f^3$
$V = 4^3$
$V = 64$

The volume is 64 cubic feet.

Let's try another example.

Example:

The figure illustrated represents a cube shaped storage box that you will use to store equipment during the training exercise. Find the volume of the storage box.



$V = f^3$
$V = 12^3$
$V = 1728$
The volume is 1728 cubic feet.

Let's test your memory. Do you recall the formula for computing the surface area? If your answer was $SA = 6(face^2)$, you're correct! How about finding the volume? If you said, $V = f^3$ you're correct again! If you had a problem using these formulas, review this section again before moving on.

b. Rectangular solids. Rectangular solids are similar to cubes except that their faces are not necessarily squares. The sum of the area of the six sides equals the surface area, so the surface area is found by adding the area of the two ends, the two sides, the top and the bottom. The formula for finding the surface area is: SA = 2(h x w) + 2(h x l) + 2(w x l). Volume is found by using the original formula that was presented in discussing the cube: V = lwh.

Example:

Find the surface area and the volume of a milvan (a milvan is large steel box) 20 feet long 8 feet wide 8 feet high.

SA = 2(h x w) + 2(h x l) + 2(w x l)
SA = 2(8 x 8) + 2(8 x 20) + 2(8 x 20)
SA = 2(64) + 2(160) + 2(160)
SA = 128 + 320 + 320
SA = 768

The surface area is 768 square feet.

V = lwh
V = 20 x 8 x 8
V = 1,280

The volume of the milvan is 1,280 cubic feet.

Now, what is the volume in cubic inches? There are 1,728 cubic inches in 1 cubic foot. To obtain cubic inches, simply multiply the total cubic feet by 1,728.

Example:

cu in = 1,728 x cu ft
cu in = 1,728 x 1,280
cu in = 2,211,840

The volume is 2,211,840 cubic inches.

Let's look at another problem.

Example:

Pvt. Layaway has a box of uniforms stored in supply. The box is 30 inches high 27 inches wide and 48 inches long. What is the volume of the box?

V = lwh
V = 48 x 27 x 30
V = 38,880

The volume is 38,880 cubic inches.

Let's change the cubic inches to cubic feet. Remember, there are 1,728 cubic inches in 1 cubic foot.

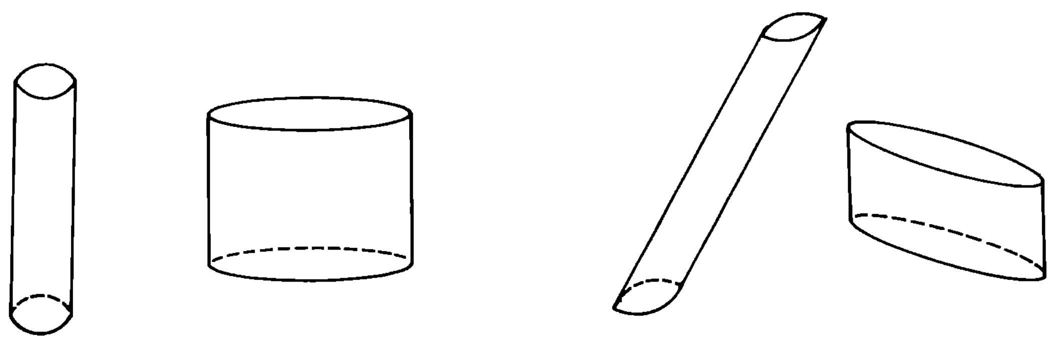
Example:

cu ft = cu in ÷ 1,728
cu ft = 38,880 ÷ 1,728
cu ft = 22.5

The volume is 22.5 cubic feet.

Remember, to compute the surface area for a rectangular solid, the formula is SA = 2(h x w) + 2(h x l) + 2(w x l). Now, what formula is used to compute the volume of a rectangular solid? That's correct, it is V = lwh. Make sure you can use these formulas before moving on.

c. Cylinder. This geometric figure is best typified by a tin can. There are two kinds of cylinders, right cylinders and oblique cylinders (fig 5-17). Since we rarely see oblique cylinders, this discussion will be limited to right cylinders.



RIGHT CYLINDERS OBLIQUE CYLINDERS

Fig 5-17. Cylinder.

A right cylinder has two equal circular bases and a side or lateral surface perpendicular to the bases. We are interested in three facts about cylinders: lateral surface area, total surface area, and volume. All three are based on principles that have been previously discussed. Lateral surface area does not include the top and bottom of the cylinder.

If the cylinder is cut vertically and opened up, it will form a rectangle with h for one dimension and $2\pi r$ (the circumference of the top or bottom) for the other. Lateral surface, then, would be equal to $2\pi rh$.

Example:

Find the lateral surface area (LSA) of a cylinder with a radius of 7 inches and a height of 14 inches.

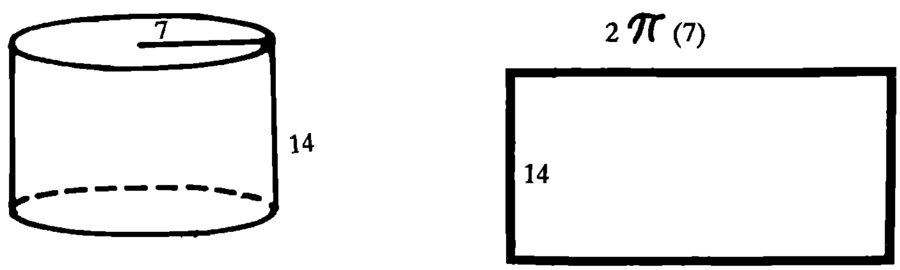
LSA = $2\pi rh$
LSA = 2 x 3.14 x 7 x 14
LSA = 615.44

The lateral surface area is 615.44 square inches.

Total surface area is equal to the lateral surface plus the area of both bases (top and bottom). The formula is TSA = $2\pi rh + 2(\pi r^2)$. Through algebraic manipulation, this can be simplified to TSA = $2\pi r(r + h)$.

Example:

Find the total surface area of a cylinder; the radius is 7 inches and the height 14 inches.



TSA = $2\pi r(r + h)$
TSA = 2 x 3.14 x 7(7 + 14)
TSA = 43.96(21)
TSA = 923.16

The total surface area is 923.16 square inches.

Now that you have found the total surface area, let's find the volume of a cylinder. The procedure is comparable to the one used to compute the volume of a rectangular solid. Remember that, when using lwh, you are finding the area (lw) of the base of a rectangle. Now, multiply the area by the number of units of the height(h). To apply this formula to cylinders, the area of the base is πr^2. Therefore, to find the volume of a cylinder use $V = \pi r^2 h$.

Example:

Find the volume of a cylinder. The radius is 7 inches and the height is 14 inches.

$V = \pi r^2 h$
$V = 3.14 \times 7^2 \times 14$
$V = 3.14 \times 49 \times 14$
$V = 2154.04$

The volume is 2154.04 cubic inches.

What is the formula used to find the lateral surface area for a cylinder? You're right, it's $LSA = 2\pi rh$. How about total surface area? Yes, it's $TSA = 2\pi r(r + h)$. Now what about the volume of a cylinder? Of course, it's $V = \pi r^2 h$.

5302. Square Root and Triangles

a. General. Back in study unit 3, in the discussion of proportion, Cpl Chainsaw discovered a method for finding the height of a tree by measuring its shadow and relating it to the height and the length of the shadow of some known object. Cpl Chainsaw used a proportion to solve his problems, and unknowingly he was applying a principle that involves the relationship between sides and angles of similar triangles. There are several relationships between triangles, particularly right triangles, that do not involve difficult mathematics but which can be of use to you in many situations. One is the earlier mentioned relationship of similar triangles; the other is the Pythagorean property. The pythagorean law or property is a basic tool in geometric and trigonometric (relating to trigonometry) calculations. It has been known since approximately 500 BC when the Greek philosopher-mathematician Pythagoras proved the relationship of the sides of a right triangle. The scuttlebutt is that one day Pythagoras was staring at a mosaic floor. The patterns so intrigued him that he started breaking them up in his mind's eye into right triangles. He noticed that when a square was constructed on each side of a triangle that the areas of the squares had a particular relationship (fig 5-18).

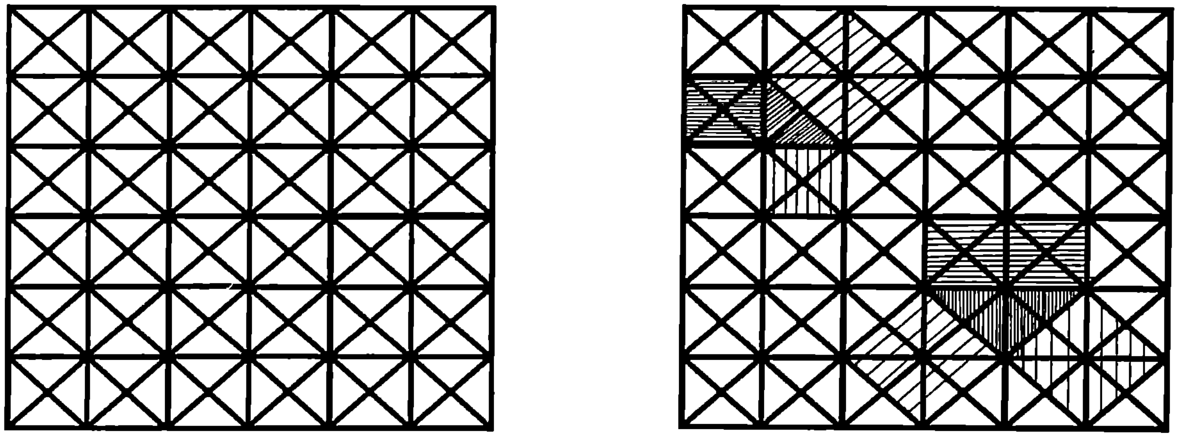


Fig 5-18. Visions of Pythagoras.

What Pythagoras noticed was that if he added the squares of the two shorter sides of a right triangle, the sum was equal to the square of the long side (hypotenuse) of the triangle. This is symbolized in an equation as $a^2 + b^2 = c^2$ where a and b are the lengths of the sides of a right triangle and c is the hypotenuse. This relationship was mentioned at the beginning of this study unit. In ancient times, there was a group of Egyptian surveyors called "rope stretchers." These Egyptian surveyors used the Pythagorean theorem applied to a piece of rope to establish right angles or corners or property. Their rope had knots tied at even intervals.

Using the relationship 3:4:5, they could form a right triangle and a perfect 90° angle (fig 5-19).

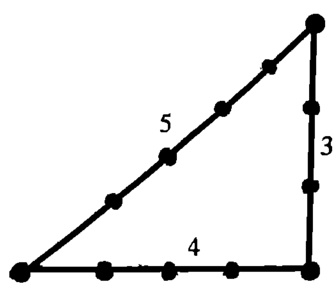


Fig 5-19. Knots in rope used to form a 90° angle.

Before you can solve problems using the Pythagorean property, you must know how to find the square root of a number. Let's see how this is done.

b. Square root. The square root of any number (x), is that number which, when multiplied by itself, gives (x). It can be found in several ways. Some square roots are used so often that they become a matter of common knowledge just as the addition and multiplication tables. For example 144 is the square of 12, which makes 12 the square root of 144 (12 x 12 = 144).

The squares of integers from 1 to 20 are used enough to be memorized. If you know the squares, then consequently you know the square root of these squares (fig 5-20).

n	n^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400

Fig 5-20. Squares of the numbers (n) from 1 - 20.

There are tables in many textbooks that give the square root of every number up to 1,000, some go even higher. When tables are not available, the computational method is used. There is no "new" method for computing square root, just a fairly rigorous method that has been used for years but needs practice to be maintained.

Example:

What is the square root of 1156?

(1) The square root symbol (√) over the number is called a radical sign.

(2) Starting at the decimal point, group the digits in pairs.

$$\sqrt{11\ 56.}$$

(3) There will be one digit in the answer for every group under the radical. Over the first group, put the largest number whose square does not exceed 11. For example, three (3) squared is 9, 4^2 is 16. The largest that can be used is 3. Square 3 and write the product under the first group.

```
  3
√11 56
  9
```

(4) Subtract the 9 from 11 and bring down the next group.

```
  3
√11 56
  9
  2 56
```

(5) Double the part of the root already found (3 doubled is 6) and add a zero to form a trial divisor. Place this next to 256.

```
     3
    √11 56
     9
60    256
```

(6) Divide 256 by the trial divisor (60) and write the quotient (4) above the second group as the <u>probable</u> second figure of the root.

```
    3  4
   √11 56
    9
60   256
```

(7) Correct the trial divisor (60) by adding to it the second number of the root (4). Then multiply the corrected divisor (64) by the second number of the root (4). Next, subtract the product from the remainder and write any remainder that's left over below it. In this case, it comes out even. The square root of 1156 is 34. This can be checked by squaring 34.

```
    3  4
   √11 56
    9
~~60~~  256
64   256
```

<u>Check</u>:

34 x 34 = 1,156

The square root of 1,156 is 34.

What happens when the square root does not come out even? Let's find out.

<u>Note</u>: Some people like to keep the "square" whole and bring down each group of numbers as needed. Your next example demonstrates this technique.

<u>Example</u>:

What is the square root of 2.

(1) We know that 2 is not a perfect square and decide to carry the calculation to three decimal places. This means that you must add three groups of zeros. The decimal point is placed directly over its original place.

```
      .
     √2.000000
```

(2) The highest number whose square does not exceed 2 is 1. The square of 1 is 1. Square 1 and write the product under the first group. Subtract 1 from 2 and bring down the next group.

```
      1.
     √2.000000
      1
      1 00
```

(3) Double the part of the root found (1 doubled is 2) and add zero to form a trial divisor of 20. Place this next to 100.

```
      1.
     √2.000000
      1
 20   100
```

(4) Divide 100 by the trial divisor 20. It goes evenly 5 times, but 5 will not work. Why? Remember, you must take the trial divisor (20) and add it to the second number of the root (5). You then multiply the corrected divisor (25) by the second number of the root (5). So, 25 x 5 = 125. This exceeds 100; therefore, 5 cannot be used as the quotient. The correct number is 4. Place this as the second figure in the root and add it to the trial divisor.

```
      1. 4
     √2.000000
      1
~~20~~ 100
 24
```

(5) Multiply the corrected divisor (24) by the second number of the root (4) and subtract the product from 100. Write any remainder left below and bring down the next group.

```
      1. 4
     √2.000000
      1
20     100
24      96
         400
```

(6) Again, double the part of the root already found (14 doubled is 28) and add a zero to form a trial divisor (280). Place this next to 400. Divide 400 by the trial divisor (280). Write the quotient (1) above the next group as the probable third figure of the root (1 x 280 = 280); this does not exceed 400. Next, correct the trial divisor (280) by adding to it the third figure of the root (1). Multiply the corrected divisor (281) by the third number of the root (1). Subtract the product (281 x 1 = 281) from the remainder and write any remainder left below.

Bring down the next group.

```
      1. 4 1
     √2.000000
      1
20     100
24      96
 280     400
 281     281
         11900
```

(7) Double the part of the root already found (141 doubled = 282) and add a zero to form a trial divisor (2820). Place this next to 11900. Divide 11900 by the trial divisor (2820). Write the quotient (4) above the next group as the probable fourth figure of the root (4 x 2820 = 11280) which does not exceed 11900. Place the 4 as the fourth figure of the root. Correct the trial divisor (2820) by adding to it the fourth figure in the root (4).

Multiply the divisor by 4.

$$
\begin{array}{rl}
 & \underline{1.\ 4\ 1\ 4} \\
 & \sqrt{2.000000} \\
 & \underline{1} \\
\not{20} & 100 \\
24 & \underline{96} \\
\not{280} & 400 \\
281 & \underline{281} \\
\not{2820} & 11900 \\
2824 & \underline{11296} \\
 & 604
\end{array}
$$

Note: You would then subtract the product (2824 x 4 = 11296) from the remainder (11900) and be left with a remainder of 604. Since you only need to find the root to three places, no further computations are necessary.

So, the square root of 2 is 1.414. It could be carried further. Keep in mind that if you square 1.414, you will not get 2. It will be close though. The more decimal places that the root is carried, the closer to 2 you will get when you square the root. For our purposes, three places is close enough.

Let's try one more example.

Example:

Find the square root of 912.04.

(1) Starting at the decimal point, group the digits in pairs. Notice that the left-most group (9) has only one digit.

Note: To group each problem with or without a space between each group is a matter of personal preference. As you become more experienced with square roots, you may decide not to place a space between each group as noted with the example below.

$$\sqrt{9\ 12.\ 04}$$

(2) Start with the first group under the radical, in this case (9). Over the first group, put the largest number whose square does not exceed 9. The highest number whose square does not exceed 9 is 3. Square the 3 and subtract it from 9.

Bring down the next group. Double the part of the root already found (3 doubled is 6) and add a zero to form a real divisor (60). Notice however, that this trial divisor (6) is too large for 12.

3
√912.04
9
60 12

(3) In this case, there is no trial divisor so you must place a zero as the second figure of the root and bring down the next group. Double the part of the root found (30 doubled is 60) and add a zero to form a trial divisor of 600. <u>This is an important step that is often overlooked</u>. Divide 1204 by the trial divisor (600). Write the quotient (2) above the next group as the probable third figure of the root. Correct the trial divisor (600) by adding to it the third figure in the root (2). Multiply the corrected divisor (602) by the third number of the root (2).

3 0. 2
√912.04
9
~~60~~ 12 04
~~600~~ 12 04
602

Subtract the product from the remainder and write any remainder left over. In this case the answer comes out evenly. The square root of 912.04 is 30.2. This can be checked by squaring 30.2.

<u>Check</u>:

30.2
x 30.2
912.04

The square root of 912.04 is 30.2.

You have just reviewed three detailed examples on how to compute the square root of any number. If you had a difficult time with any of these examples, review this section before moving on to right triangles.

c. <u>Unknown side of triangles.</u> Now that we have discussed the principles of the pythagorean property and the computation of square roots, let's put them together to solve problems involving the sides of right triangles. Just for review, the pythagorean property stated in words is: The sum of the squares of two sides of a right triangle equals the square of the hypotenuse.

Example:

Two sides of a right triangle are 9 inches (a) and 12 inches (b). Find the length of the hypotenuse (third side).

$$a^2 + b^2 = c^2$$
$$9^2 + 12^2 = c^2$$
$$81 + 144 = c^2$$
$$225 = c^2$$

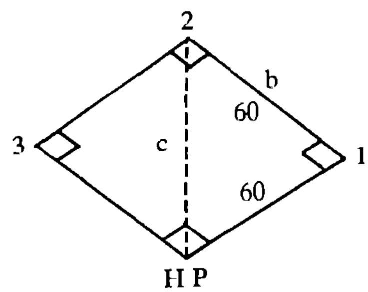
Note: To find c you must take the square root of c^2.

$$225 = c^2$$
$15 = c =$ The hypotenuse is 15 inches.

Let's try another example.

Example:

The illustration below represents a softball field. What is the distance from home plate (HP) to second base (2)? The bases are 60 feet apart.



$$a^2 + b^2 = c^2$$
$$60^2 + 60^2 = c^2$$
$$3600 + 3600 = c^2$$
$$7200 = c^2$$
$$\sqrt{7200} = c$$
$$84.85 = c$$

The distance from home plate to second base is 84.85 feet.

The last example shows you that if the hypotenuse of a right triangle is known, the unknown side can be found. Let's see how this is done.

Example:

The hypotenuse of a right triangle is 13 feet and one side is 5 feet. Find the third side.

$$a^2 + b^2 = c^2$$
$$a^2 + 5^2 = 13^2$$
$$a^2 + 25 = 169$$
$$a^2 = \sqrt{144}$$
$$a = 12$$

The third side is 12 feet long.

You have just used the proper formulas to compute unknown distances of right triangles. Next, you will solve unknown dimensions of similar triangles.

d. Similar triangles. If the corresponding angles of two triangles are equal, the triangles are said to be similar. The triangles do not have to be the same size or have sides the same length; only the angles need to be the same (fig 5-21).

Fig 5-21. Similar triangles.

In similar triangles, corresponding sides are proportional. If two sides of one triangle and one side of another triangle are known, the length of the missing corresponding side can be found. This can be illustrated by referring to the problem concerning the height of the tree in study unit 3.

Example:

In the illustration provided (fig 5-22), the tree and the individual measured are assumed to be at right angles to the ground. The sun hits both the tree and individual at the same angle. Since the angles are the same measure, the sides are proportional. The lengths of the shadows correspond to the sides of a triangle. The length of the other side is the height of the object (tree or individual). Therefore, a proportion is set up and you can find the missing dimension. To restate, if corresponding angles of two triangles are equal, the triangles are similar and the corresponding sides are proportional.

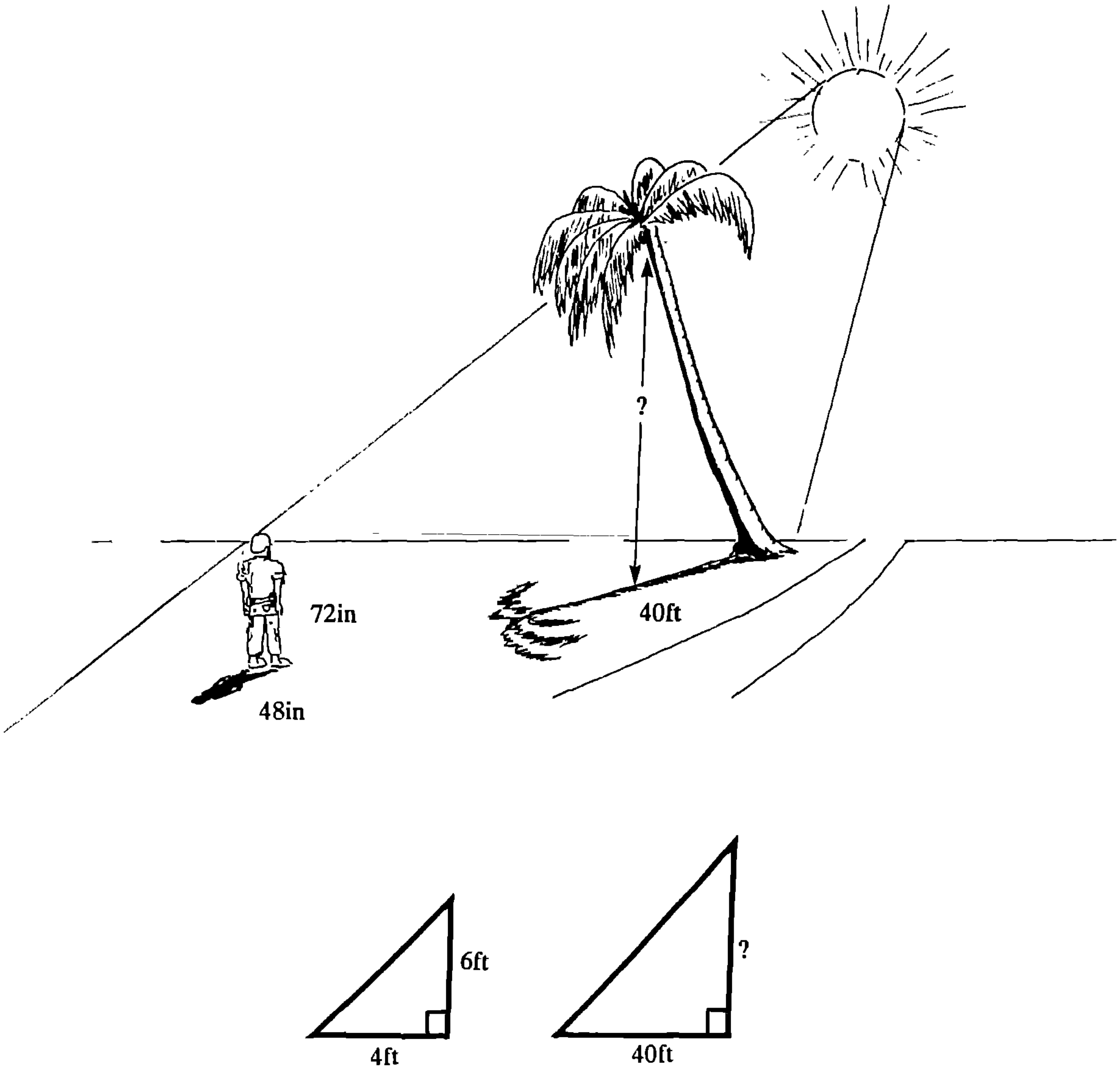


Fig 5-22. Illustration of similar triangles.

$$\frac{4}{40} = \frac{6}{x}$$

$$4x = 240$$

$$\frac{4x}{4} = \frac{240}{4}$$

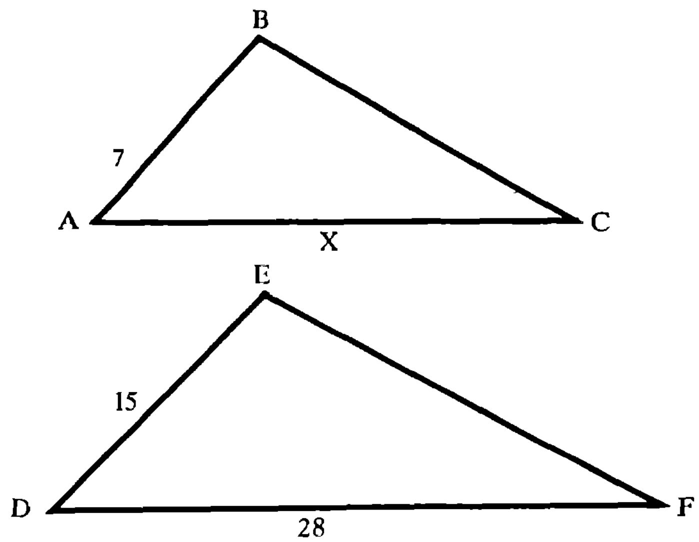
$$x = 60$$

The tree is 60 feet tall.

Let's work another problem that's a little more difficult.

Example:

Find side x if triangle ABC is similar to triangle DEF.



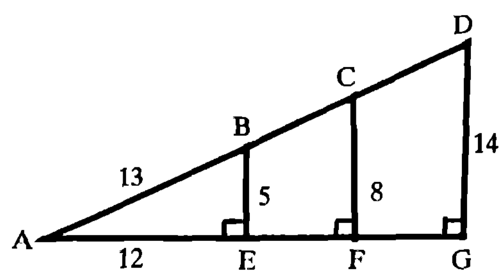
Triangle ABC		Triangle DEF
$\frac{7}{x} = \frac{15}{28}$		$\frac{7}{15} = \frac{x}{28}$
$15x = 196$	or	$15x = 196$
$\frac{15x}{15} = \frac{196}{15}$		$\frac{15x}{15} = \frac{196}{15}$
$x = 13\ 1/15$		$x = 13\ 1/15$

Side x equals 13 1/15.

Let's try another problem that's a little more difficult.

Example:

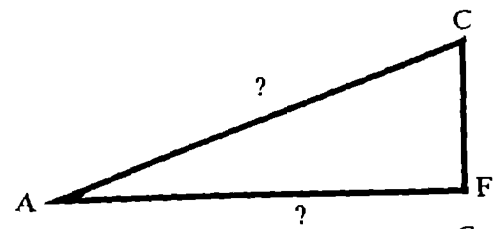
Find the missing sides in triangle ACF and triangle ADG.



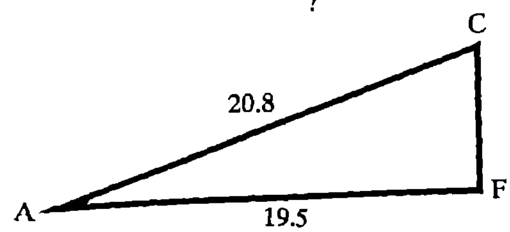
Since angle A is part of each triangle and each has a right angle, the third angle also must be equal. Therefore the triangles are similar.

The three sides of triangle ABE are given as are one side of triangle ACF and triangle ADG, so proportions can be set up to find the missing sides.

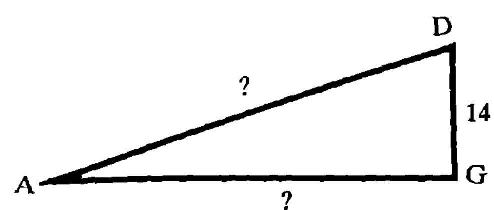
$$\frac{13}{5} = \frac{AC}{8}$$
$$5AC = 104$$
$$\frac{5AC}{5} = \frac{104}{5}$$
$$AC = 20.8$$



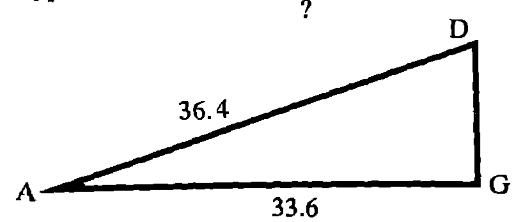
$$\frac{12}{5} = \frac{AF}{8}$$
$$5AF = 96$$
$$\frac{5AF}{5} = \frac{96}{5}$$
$$AF = 19.2$$



$$\frac{13}{5} = \frac{AD}{14}$$
$$5AD = 182$$
$$\frac{5AD}{5} = \frac{182}{5}$$
$$AD = 36.4$$



$$\frac{12}{5} = \frac{AG}{14}$$
$$5AG = 168$$
$$\frac{5AG}{5} = \frac{168}{5}$$
$$AG = 33.6$$

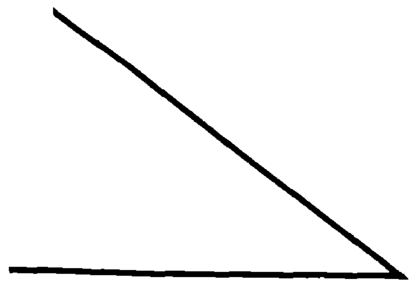


As you can see, finding the missing sides is easy using proportions.

Lesson Summary. In this lesson you calculated the surface area and volume of a cube, rectangular solid, and cylinder. You also used the proper operations to find the square root of a number, the unknown side of right triangles as well as the length of triangles. You should now be ready to complete the unit exercise.

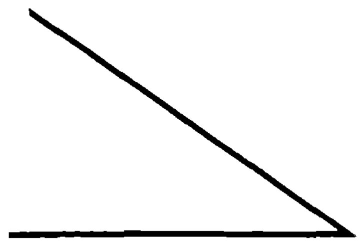
Unit Exercise: Complete items 1 through 64 by performing the action required. Check your responses against those listed at the end of this study unit.

1. Using your protractor, measure the angle to obtain its measurement in degrees.



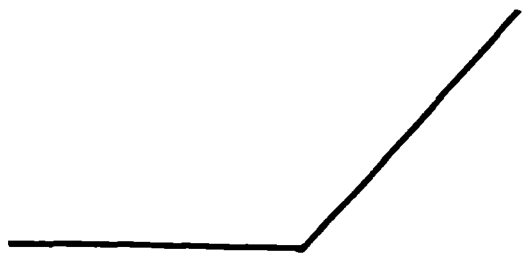
a. 25°
b. 27°
c. 35°
d. 37°

2. Using your protractor, measure the angle to obtain its measurement in degrees.



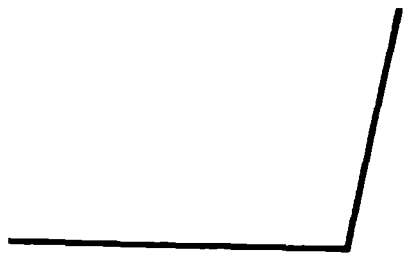
a. 30°
b. 35°
c. 40°
d. 45°

3. Using your protractor, measure the angle to obtain its measurement in degrees.



a. 130°
b. 135°
c. 140°
d. 145°

4. Using your protractor, measure the angle to obtain its measurement in degrees.

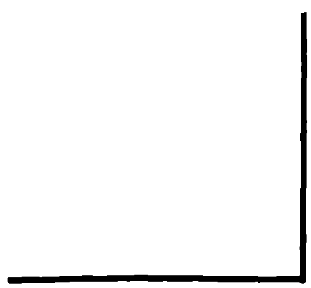


a. 90°
b. 100°
c. 110°
d. 1150°

5. Using your protractor, measure the angle to obtain its measurement in degrees.

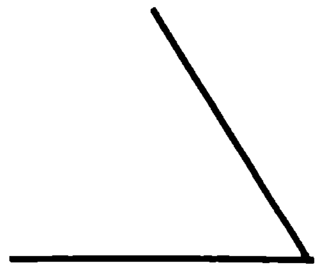
a. 90°
b. 180°
c. 270°
d. 360°

6. Using your protractor, measure the angle to obtain its measurement in degrees.



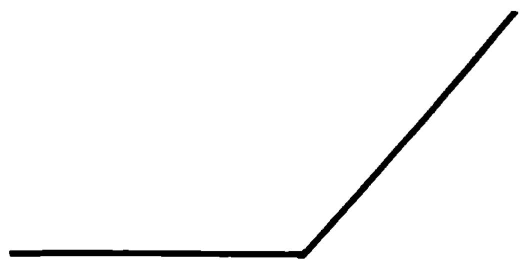
a. 85°
b. 90°
c. 95°
d. 100°

7. Classify the angle shown.



a. Right Angle
b. Obtuse Angle
c. Acute Angle
d. Straight Angle

8. Classify the angle shown.



a. Right Angle
b. Obtuse Angle
c. Acute Angle
d. Straight Angle

9. Classify the angle shown.

a. Right Angle
b. Obtuse Angle
c. Acute Angle
d. Straight Angle

10. Classify the angle shown.

a. Right Angle
b. Obtuse Angle
c. Acute Angle
d. Straight Angle

11.
$$\begin{array}{r} 18^\circ\ 4' \\ \underline{+\ 180^\circ} \end{array}$$

a. $188^\circ\ 4'$
b. $189^\circ\ 4'$
c. $198^\circ\ 4'$
d. $199^\circ\ 4'$

12.
$$\begin{array}{r} 210^\circ\ 7' \\ \underline{-\ 180^\circ} \end{array}$$

a. $30^\circ\ 7'$
b. $31^\circ\ 7'$
c. $32^\circ\ 7'$
d. $33^\circ\ 7'$

13.
$$\begin{array}{r} 38^\circ\ 48' \\ \underline{+\ 180^\circ\ 26'} \end{array}$$

a. $218^\circ\ 14'$
b. $218^\circ\ 48'$
c. $219^\circ\ 14'$
d. $219^\circ\ 48'$

14.
$$\begin{array}{r} 274^\circ\ 34' \\ \underline{-\ 180^\circ\ 44'} \end{array}$$

a. $94^\circ\ 34'$
b. $94^\circ\ 50'$
c. $93^\circ\ 34'$
d. $93^\circ\ 50'$

15.
$$\begin{array}{r} 118^\circ\ 29'\ 29'' \\ \underline{+\ 140^\circ\ 42'\ 24''} \end{array}$$

a. $249^\circ\ 1'\ 53''$
b. $249^\circ\ 11'\ 53''$
c. $259^\circ\ 1'\ 53''$
d. $259^\circ\ 11'\ 53''$

16.
$$\begin{array}{r} 124^\circ\ 17'\ 35'' \\ \underline{-\ 82^\circ\ 18'\ 34''} \end{array}$$

a. $41^\circ\ 59'\ 1''$
b. $41^\circ\ 55'\ 11''$
c. $42^\circ\ 59'\ 1''$
d. $42^\circ\ 55'\ 11''$

17.
$$\begin{array}{r} 60^\circ\ 14'\ 2'' \\ \underline{+\ 18^\circ\ 27'\ 17''} \end{array}$$

a. $77^\circ\ 14'\ 19''$
b. $77^\circ\ 41'\ 19''$
c. $78^\circ\ 41'\ 19''$
d. $78^\circ\ 47'\ 19''$

18.
$$\begin{array}{r} 25^\circ\ 16'\ 7'' \\ -\ \underline{14^\circ\ \ \ \ \ \ \ 8''} \end{array}$$

a. 12° 15' 59" c. 10° 15' 59"
b. 11° 15' 59" d. 9° 15' 59"

19. Find the perimeter of a square that has a side of 48 inches.

a. 2,304 sq in c. 129 in
b. 2,403 sq in d. 192 in

20. Find the area of a square that has a side of 48 inches.

a. 2,403 sq in c. 192 in
b. 2,304 sq in d. 129 in

21. Find the perimeter of a square that has a side of 16.5 inches.

a. 272.25 sq in c. 66 in
b. 227.25 sq in d. 63 in

22. Find the area of a square that has a side of 16.5 inches.

a. 227.25 sq in c. 63 in
b. 272.25 sq in d. 66 in

23. Find the perimeter of a rectangle with a length of 18 inches and a width of 12 inches.

a. 214 sq in c. 50 in
b. 216 sq in d. 60 in

24. Find the area of a rectangle that has a length of 18 inches and a width of 12 inches.

a. 214 sq in c. 50 in
b. 216 sq in d. 60 in

25. Find the perimeter of a rectangle that has a length of 52 inches and a width of 39 inches.

a. 2,694 sq in c. 128 in
b. 2,496 sq in d. 182 in

26. Find the area of a rectangle that has length of 52 inches and a width of 39 inches.

a. 2,028 sq in c. 182 in
b. 1,694 sq in d. 128 in

27. Find the perimeter of a parallelogram with a width of 12 inches and a base of 24 inches.

a. 298 sq in
b. 288 sq in
c. 84 in
d. 72 in

28. Find the area of a parallelogram with a base of 24 inches and a height of 12 inches.

a. 288 sq in
b. 268 sq in
c. 36 in
d. 84 in

29. Find the perimeter of a parallelogram with a width of 15.3 inches and a base of 50.2 inches.

a. 768.06 sq in
b. 786.06 sq in
c. 130 in
d. 131 in

30. Find the area of a parallelogram with a width of 15.3 inches and a base of 50.2 inches.

a. 786.06 sq in
b. 768.06 sq in
c. 131 in
d. 130 in

31. Find the perimeter of a trapezoid that measures 10 feet by 6 feet by 15 feet by 6 feet.

a. 250 sq ft
b. 200 sq ft
c. 53 ft
d. 37 ft

32. Find the area of a trapezoid that has a base 1 (b1) of 10 feet, base 2 (b2) of 15 feet and height (h) of 7 feet.

a. 87.5 sq ft
b. 85.7 sq ft
c. 25 ft
d. 20 ft

33. Find the perimeter of a trapezoid that measures 600 feet by 900 feet by 600 feet by 1200 feet.

a. 630,000 sq ft
b. 620,000 sq ft
c. 3200 ft
d. 3300 ft

34. Find the area of a trapezoid has a base 1 (b1) of 900 feet, a base 2 (b2) of 1200 feet, and a height (h) of 500 feet.

a. 552,000 sq ft
b. 525,000 sq ft
c. 2600 ft
d. 2500 ft

35. Find the perimeter of a triangle that measures 20 feet by 20 feet by 28 feet.

a. 208 sq in
b. 280 sq in
c. 86 in
d. 68 in

36. Find the area of a triangle that has a base of 20 feet and a height of 15 feet.

a. 150 sq ft
b. 160 sq ft
c. 35 ft
d. 300 ft

37. Find the perimeter of a triangle shaped concrete platform that measures 40 feet by 80 feet by 40 feet.

a. 3200 sq ft
b. 3300 sq ft
c. 160 ft
d. 150 ft

38. Find the area of a triangle shaped concrete platform that has a base of 80 feet and a height of 38 feet.

a. 1,250 sq ft
b. 1,520 sq ft
c. 118 ft
d. 181 ft

39. Find the circumference of a anti-tank landmine that has a radius of 7.5 inches.

a. 53.1 in
b. 51.1 in
c. 49.1 in
d. 47.1 in

40. Find the circumference of a fuel can that has a diameter of 19 inches.

a. 56.66 in
b. 57.66 in
c. 58.66 in
d. 59.66 in

41. The illumination radius for the M49A1 trip flare is 300m. When activated, how much area is illuminated?

a. 228,600 sq m
b. 282,600 sq m
c. 22,860 sq m
d. 28,260 sq m

42. The illumination diameter for the MK1 illumination hand grenade is 200m. When activated, how much area is illuminated?

a. 125,600 sq m
b. 152,600 sq m
c. 31,400 sq m
d. 34,100 sq m

43. The radius of a chemically contaminated area is 550m. How much area is chemically contaminated?

a. 994,850 sq m
b. 949,850 sq m
c. 949,580 sq m
d. 949,085 sq m

44. The radius that the LVS (Logistics Vehicle System) makes when turning in a circle is 38.5 feet. How much area does the LVS need when making a complete circle?

a. 46,542.65 sq ft
b. 46,452.65 sq ft
c. 4,654.265 sq ft
d. 4,564.265 sq ft

45. Find the surface area of a company supply box that is 8 feet wide, 8 feet high, and 8 feet long.

a. 384 sq ft
b. 348 sq ft
c. 512 cu ft
d. 521 cu ft

46. Find the volume of a company supply box that measures 8 feet wide, 8 feet high, and 8 feet long.

a. 348 sq ft
b. 384 sq ft
c. 521 cu ft
d. 512 cu ft

47. Find the surface area of a gas chamber that measures 11 feet wide, 11 feet high, and 11 feet long.

a. 762 sq ft
b. 726 sq ft
c. 1,331 cu ft
d. 1,313 cu ft

48. Find the volume of a gas chamber that measures 11 feet wide, 11 feet high, and 11 feet long.

a. 726 sq ft
b. 762 sq ft
c. 1,313 cu ft
d. 1,331 cu ft

49. Find the surface area of a milvan that measures 8 feet wide, 8 feet high, and 10 feet long.

a. 448 sq ft
b. 484 sq ft
c. 604 cu ft
d. 640 cu ft

50. Find the volume of a milvan that measures 8 feet wide, 8 feet high, and 10 feet long.

a. 484 sq ft
b. 448 sq ft
c. 640 cu ft
d. 604 cu ft

51. Find the surface area of a concrete bunker that measures 6 feet high, 6 feet wide, and 10 feet long.

a. 312 sq ft
b. 321 sq ft
c. 306 cu ft
d. 360 cu ft

52. Find the volume of a concrete bunker that measures 6 feet high, 6 feet wide, and 10 feet long.

a. 321 sq ft
b. 312 sq ft
c. 360 cu ft
d. 306 cu ft

53. Find the volume of a cylindrical water tank that has a radius of 9.5 feet and a height of 40 feet.

a. 2,386.4 cu ft
b. 2,836.4 cu ft
c. 11,335.4 cu ft
d. 11,533.4 cu ft

54. Find the volume of an anti-tank landmine that has a radius of 6.5 inches and a height of 4 inches.

a. 163.28 cu in
b. 136.28 cu in
c. 530.66 cu in
d. 503.66 cu in

55. Find the square root of 1,038.

a. 23.12
b. 23.21
c. 32.12
d. 32.21

56. Find the square root of 511,225.

a. 715
b. 716
c. 717
d. 718

57. Find the square root of 33.

a. 4.64
b. 4.74
c. 5.64
d. 5.74

58. Find the square root of 94.8676.

a. 9.64
b. 9.74
c. 9.84
d. 9.94

59. Find the missing side (c) of a right triangle that has a side (a) of 5 inches and a side (b) of 6 inches.

a. 6.88 in
b. 7.81 in
c. 7.91 in
d. 7.98 in

60. Find the missing side (c) of a right triangle that has a side (a) of 3 inches and a side (b) of 9 inches.

a. 9.48 in
b. 9.58 in
c. 9.68 in
d. 9.85 in

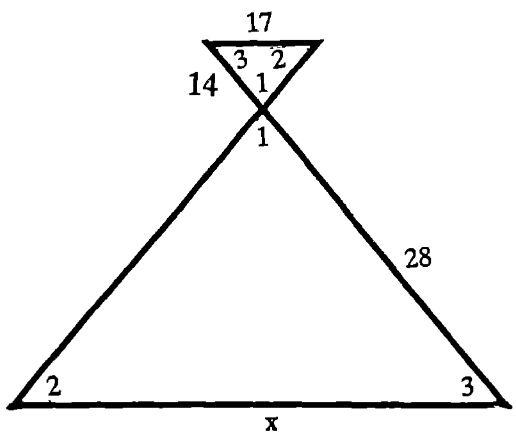
61. Find the missing side (b) of a right triangle that has a side (a) of 24 inches and a side (c) of 26 inches.

a. 9 in
b. 10 in
c. 11 in
d. 12 in

62. Find the missing side (a) of a right triangle that has a side (b) of 15 inches and a side (c) of 39 inches.

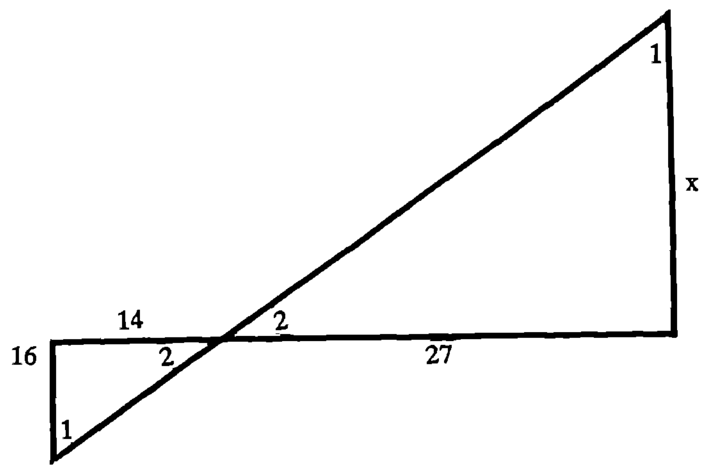
a. 33 in
b. 34 in
c. 35 in
d. 36 in

63. Find the missing side (x) of the triangle illustrated.



a. 30
b. 32
c. 34
d. 36

64. Find the missing side (x) of the triangle illustrated.



a. 30.58
b. 30.85
c. 31.58
d. 31.85

UNIT SUMMARY

This study unit provided practical applications of geometric principles on angles, plane figures, solid figures, and triangles. Additionally, you were taught, through the use of several examples, how these principles are used by Marines.

Exercise Solutions

Exercise	Answer	Reference
1.	d.	5101
2.	b.	5101
3.	a.	5101
4.	b.	5101
5.	b.	5101
6.	b.	5101
7.	c.	5102
8.	b.	5102
9.	a.	5102
10.	d.	5102
11.	c. $\begin{array}{r} 18^\circ\ 4' \\ +\ 180^\circ \\ \hline 198^\circ\ 4' \end{array}$	5103
12.	a. $\begin{array}{r} 210^\circ\ 7' \\ -\ 180^\circ \\ \hline 30^\circ\ 7' \end{array}$	5103
13.	c. $\begin{array}{r} 38^\circ\ 48' \\ +\ 180^\circ\ 26' \\ \hline 218^\circ\ 74' \end{array} = 219^\circ\ 14'$	5103
14.	d. $\begin{array}{r} 274^\circ\ 34' \\ -\ 180^\circ\ 44' \\ \hline \end{array} = \begin{array}{r} 273^\circ\ 94' \\ -\ 180^\circ\ 44' \\ \hline 93^\circ\ 50' \end{array}$	5103
15.	d. $\begin{array}{r} 118^\circ\ 29'\ 29'' \\ +\ 140^\circ\ 42'\ 24'' \\ \hline 258^\circ\ 71'\ 53'' \end{array} = 259^\circ\ 11'\ 53''$	5103
16.	a. $\begin{array}{r} 124^\circ\ 17'\ 35'' \\ -\ 82^\circ\ 18'\ 34'' \\ \hline \end{array} = \begin{array}{r} 123^\circ\ 77'\ 35'' \\ -\ 82^\circ\ 18'\ 34'' \\ \hline 41^\circ\ 59'\ 1'' \end{array}$	5103
17.	c. $\begin{array}{r} 60^\circ\ 14'\ 2'' \\ +\ 18^\circ\ 27'\ 17'' \\ \hline 78^\circ\ 41'\ 19'' \end{array}$	5103
18.	b. $\begin{array}{r} 25^\circ\ 16'\ 7'' \\ -\ 14^\circ\qquad 8'' \\ \hline \end{array} = \begin{array}{r} 25^\circ\ 15'\ 67'' \\ -\ 14^\circ\qquad 8'' \\ \hline 11^\circ\ 15'\ 59'' \end{array}$	5103

		Reference
19. d.	$P = 4s$ $P = 4(48)$ $P = 192$ in	5201
20. b.	$A = s^2$ $A = (48)^2$ $A = 2{,}304$ sq in	5201
21. c.	$P = 4s$ $P = 4(16.5)$ $P = 66$ in	5201
22. b.	$A = s^2$ $A = (16.5)^2$ $A = 272.25$ sq in	5201
23. d.	$P = 2l + 2w$ $P = 2(18) + 2(12)$ $P = 36 + 24$ $P = 60$ in	5202
24. b.	$A = lw$ $A = 18 \times 12$ $A = 216$ sq in	5202
25. d.	$P = 2l + 2w$ $P = 2(52) + 2(39)$ $P = 104 + 78$ $P = 182$ in	5202
26. a.	$A = lw$ $A = 52 \times 39$ $A = 2{,}496$ sq in	5202
27. d.	$P = 2b + 2w$ $P = 2(24) + 2(12)$ $P = 48 + 24$ $P = 72$ in	5203
28. a.	$A = bh$ $A = (24)(12)$ $A = 288$ sq in	5203
29. d.	$P = 2b + 2w$ $P = 2(50.2) + 2(15.3)$ $P = 100.4 + 30.6$ $P = 131$ in	5203
30. b.	$A = bh$ $A = (50.2)(15.3)$ $A = 768.06$	5203

		Reference
31. d.	$P = a + b + c + d$ $P = 10 + 6 + 15 + 6$ $P = 37ft$	5204
32. a.	$A = 1/2h(b1 + b2)$ $A = 1/2(7)(10 + 15)$ $A = 3.5(25)$ $A = 87.5$ sq ft	5204
33. d.	$P = a + b + c + d$ $P = 600 + 900 + 600 + 1200$ $P = 3300$ ft	5204
34. b.	$A = 1/2h(b1 + b2)$ $A = 1/2(500)(900 + 1200)$ $A = 250(2100)$ $A = 525,000$ sq ft	5204
35. d.	$P = a + b + c$ $P = 20 + 20 + 28$ $P = 68$ in	5205
36. a.	$A = 1/2bh$ $A = 1/2(20 \times 15)$ $A = 1/2(300)$ $A = 150$ sq ft	5205
37. c.	$P = a + b + c$ $P = 40 + 80 + 40$ $P = 160$ ft	5205
38. b.	$A = 1/2bh$ $A = 1/2(80 \times 38)$ $A = 1/2(3040)$ $A = 1520$ sq ft	5205
39. d.	$C = 2\pi r$ $C = 2(3.14)7.5$ $C = 47.1$ in	5206
40. d.	$C = \pi d$ $C = 3.14(19)$ $C = 59.66$ in	5206
41. b.	$A = \pi r^2$ $A = 3.14(300^2)$ $A = 3.14(9000)$ $A = 282,600$ sq m	5206
42. c.	$A = \pi r^2$ $A = 3.14(100^2)$ $A = 3.14(10,000)$ $A = 31,400$ sq m	5206

Reference

43. b. $A = \pi r^2$ 5206
$A = 3.14(550^2)$
$A = 3.14(302.500)$
$A = 949,850$ sq m

44. c. $A = \pi r^2$ 5206
$A = 3.14(38.5^2)$
$A = 3.14(1,482.25)$
$A = 4,654.265$ sq ft

45. a. $SA = 6(face^2)$ 5301
$SA = 6(11^2)$
$SA = 6(121)$
$SA = 384$ sq ft

46. d. $V = f^3$ 5301
$V = 8^3$
$V = 512$ cu ft

47. b. $SA = 6(face^2)$ 5301
$SA = 6(11^2)$
$SA = 6(121)$
$SA = 726$ sq ft

48. d. $V = f^3$ 5301
$V = 11^3$
$V = 1,331$ cu ft

49. a. SA = 2(h x w) + 2(h x l) + 2(w x l) 5301
SA = 2(8 x 8) + 2(8 x 10) + 2(8 x 10)
SA = 2(64) + 2(80) + 2(80)
SA = 128 + 160 + 160
SA = 448 sq ft

50. c. V = lwh 5301
V = 10 x 8 x 8
V = 640 cu ft

51. a. SA = 2(h x w) + 2(h x l) + 2(w x l) 5301
SA = 2(6 x 6) + 2(6 x 10) + 2(6 x 10)
SA = 2(36) + 2(60) + 2(60)
SA = 72 + 120 + 120
SA = 312 sq ft

52. c. V = lwh 5301
V = 10 x 6 x 6
V = 360 cu ft

53. c. $V = \pi r^2 h$ 5301
$V = 3.14(9.5^2)40$
$V = 3.14(90.25)40$
$V = 11,335.4$ cu ft

Reference

54. c. $V = \pi r^2 h$ — 5301

$V = 3.14(6.5^2)4$

$V = 3.14(42.25)4$

$V = 530.66$ cu in

55. d. — 5302

```
       3 2. 2  1
     √1038.00 00
       9
  ~~60~~ 138
   62 124
 ~~640~~   14 00
 642   12 84
 ~~6440~~   1 16 00
 6441      64 41
           51 59
```

56. a. — 5302

```
       7 1 5.
     √511225.
      49
 ~~140~~ 212
 141 141
 ~~1420~~ 7125
 1425 7125
         0
```

57. d. — 5302

```
       5. 7  4
     √33.00 00
      25
 ~~100~~ 8 00
 107 7 49
 ~~1140~~  51 00
 1144  45 76
        5 24
```

58. b. — 5302

```
       9. 7 4
     √94.8676
      81
 ~~180~~ 13 86
 187 13 09
 ~~1940~~   7776
 1944   7776
           0
```

Reference

59. b. $a^2 + b^2 = c^2$ 5302

$5^2 + 6^2 = c^2$

$25 + 36 = c^2$

$61 = c^2$

$\sqrt{61} = c$

```
      7. 8  1
    √61.00 00
      49
~~140~~ 12 00
148  11 84
~~1560~~   16 00
1561   15 61
          39
```

7.81 in = c

60. a. $a^2 + b^2 = c^2$ 5302

$3^2 + 9^2 = c^2$

$9 + 81 = c^2$

$90 = c^2$

$\sqrt{90} = c$

```
      9. 4  8
    √90.00 00
      81
~~180~~  9 00
184   7 36
~~1880~~ 1 64 00
1888 1 51 04
       12 96
```

9.48 in = c

61. b. $a^2 + b^2 = c^2$ 5302

$24^2 + b^2 = 26^2$

$567 + b^2 = 676$

$b^2 = 100$

$b = \sqrt{100}$

```
     1 0.
    √100.
     1
  20  0
```

b = 10 in

62. d. $a^2 + b^2 = c^2$ 5302

$a^2 + 15^2 = 39^2$

$a^2 + 225 = 1521$

$a^2 = 1296$

$a = \sqrt{1296}$

```
     3 6.
    √1296.
      9
~~60~~ 396
66 396
       0
```

a = 36 in

Reference

63. c. $\frac{14}{17} = \frac{28}{x}$ 5302

$14x = 476$

$\frac{14x}{14} = \frac{476}{14}$

$x = 34$

64. b. $\frac{16}{14} = \frac{x}{27}$ 5302

$14x = 432$

$\frac{14x}{14} = \frac{432}{14}$

$x = 30.85$

STUDY UNIT 6

PROBLEM SOLVING

Introduction. When it comes to mathematical problems, the most difficult type seems to be word problems. Part of this difficulty is because of the "fear" associated with them. As you have probably noticed, this course has used word problems throughout each study unit, and, up until now, the problems have been fairly simple. This study unit is designed to reduce the fear by giving you helpful hints and a plan designed to prepare you for solving more difficult problems. You will solve word problems by employing a logical plan and translating English expressions into mathematical symbols.

Lesson 1. THE LOGICAL PLAN AND LANGUAGES

LEARNING OBJECTIVES

1. Select the five steps of the logical plan used in solving word problems.

2. Given English expressions, translate the expressions into mathematical symbols.

6101. General

Just about any successful endeavor has a sound plan to go with it. If you want to build an airfield, wire a camp, clear a village, lay a minefield, teach a lesson, take a hill, or in your case, solve a word problem, a plan will aid in the completion of the project. Let's take a look at a simple five step plan that will help you do just that!

6102. The Five Step Plan

The plan for solving problems may involve any amount of steps depending on how the operations are combined. Keep in mind that many problems can be solved by trial and error with arithmetic, much the same as the ancient Egyptians did. In most (but not all) problems, this trial and error aspect can be eliminated by applying a simple equation or formula to the information that is known. This will be an integral part of our plan. Our plan will consist of five steps; let's take a look at them.

a. Step 1 - Read the problem. This is self-explanatory. It would seem obvious, but many mistakes are made because Marines skip over important aspects in the problem. Don't read into the problem and create things that are not there; simply read the problem and stick to the facts. In real-life situations, where the problems may not be written, write all of the facts down so that the complete problem can be analyzed.

b. Step 2 - Determine the unknowns and represent them. You will have certain information given to you about a problem. There is usually a primary question asked. You must determine the method of going from this information to the solution. If the problem involves a formula or equation, there are unknown items that must be represented. Use any device that you can think of to aid you in this. Common aids that are employed are diagrams and charts. If you recall, aids have been widely used in examples throughout this course.

c. Step 3 - Write the equation. If the problem involves an equation, use the information from the preceding step. If it involves a formula, substitute the known values for the appropriate letters of the formula. If it can be solved with arithmetic, decide which operations are needed. Remember, an equation separates mathematical symbols into left and right sides with an equal sign while a formula is a mathematical rule, principle, or statement.

d. Step 4 - Solve the equation. Solve and check the formula or equation or perform the arithmetic necessary. One of the most important parts is checking or verifying that your answer is correct. Do not neglect to check!

e. Step 5 - Answer the question. The answer to an equation, a formula, or an arithmetic operation may not be the answer you are looking for. Be sure to express your answer in the terms of the original problem. You may want to know square feet, but your equation has given you an answer in square inches, so don't forget to convert.

You should see that these five steps provide a logical plan. Before you can use this plan, you must make sure that you understand the language for solving word problems. Some of the language you will cover will be a review; nevertheless, all of it will help you solve problems. But, before we study the language, can you name the five steps in the logical plan? If your answers were: read the problem, determine the unknowns and represent them, write an equation, solve the equation, and answer the question, you're correct! If not, review this section before moving on.

6103. The Language

In solving a problem, your initial job will be to translate English phrases into the language of mathematics. This is a matter of associating words with mathematical symbols. Let's see how this is done.

a. Addition. The operation of addition, symbolized by the plus sign (+), can take the place of: more than, exceeds, increased by, and the sum of.

> Example:
> The sum of six and nineteen is written 6 + 19.

b. Subtraction. The operation of subtraction, symbolized by the minus sign (-), can take the place of: the difference between, less than, decreased by, and diminished by.

> Example:
> The difference between seventeen and two is written 17 - 2.

c. Multiplication. The operation of multiplication, symbolized by the times sign (x), can take the place of: times and the product of.

> Example:
> The product of two and nine is written 2 x 9.

d. Division. The operation of division symbolized, by the divided by sign (÷), can take the place of the quotient of.

> Example:
> The quotient of m divided by seven is written m ÷ 7.

Note: Often a combination of operation signs and signs of groupings is used. Let's look at some examples.

Example:

The statement six more than the difference of five and two could be written (5 - 2) + 6. Although the use of parentheses makes it clearer, it actually is not needed because of the order of operations.

Example:

The sum of a and b, divided by 4 is written $\frac{a + b}{4}$.

Example:

One-third of the product of x and y is written $\frac{xy}{3}$.

Example:

One divided by twice the sum of a and b is written $\frac{1}{2(a + B)}$.

Example:

The product of 17 and 3 divided by 2 times their difference is written $\frac{17 \times 3}{2(17 - 3)}$.

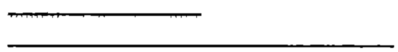
Note: We are not asking for solutions to the statements, just translations. As with most mathematics, practice is required if you are to understand the translations.

Now that you have associated words with mathematical symbols, let's look at some problem situations that involve the use of language. Diagrams will be used to help depict some of the situations.

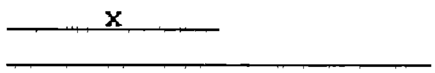
Example:

An airfield has two runways. One is twice as long as the other. Draw lines to represent the runways and label each in terms of the same variable.

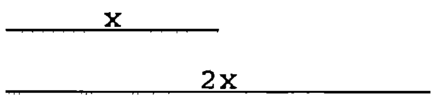
This could be done two ways. First, draw the lines.



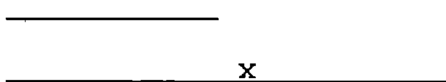
Now, to label them you must select some convenient representation for either line. Let's say that you call the smaller runway x distance in length.



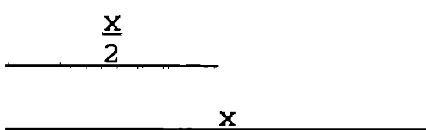
If the shorter one is x and the longer one is twice as long, then it would be 2x.



Suppose that you had labeled the longer one as x.



If it is twice as long as the shorter one, then the shorter one is 1/2x or x/2.



As you can see from this last example, some of the problems can be expressed in different ways. Let's look at four more examples.

Example:

The floor of a room is made of 90 boards laid side by side. If w equals the number of inches in the width of one board, express the width of the room.

If one board is w, then 90 boards must be 90 times w or 90w.

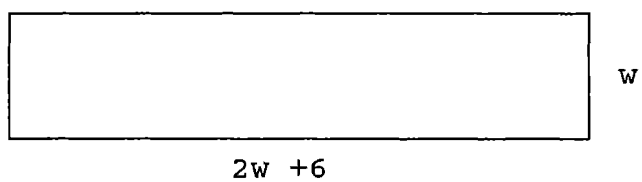
Example:

A company produced h pairs of jungles boots during its first month. Production increased by 5000 pairs each succeeding month. Express the number of pairs produced during the third month. This can be shown quite handily in chart form.

1st month	2d month	3d month
h	h + 5000	(h + 5000) + 5000

Example:

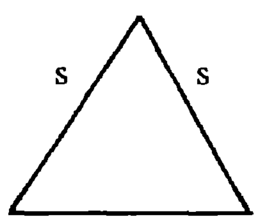
A hut is 6 feet longer than twice its width. Represent the number of feet in the width and the length. The figure is a rectangle. You can start by letting w equal the width. The length, then, is 6 added to twice this width, or 2w + 6.



Example:

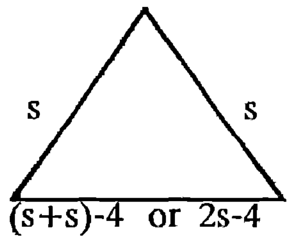
The base of a triangle with two equal sides is 4 feet less than the sum of the two equal sides. Translate this English expression into a mathematical symbol and represent the length of each side, the base, and the perimeter.

Draw the figure.



Example--continued:

By the way, what kind of triangle is it? You should have said that it is an isosceles triangle. Represent the length of each side by labeling these two equal sides with the same letter. The base is 4 feet less than the sum of these two sides.



The perimeter or the sum of the three sides is represented as:

$$s + s + 2s - 4$$
or
$$4s - 4$$

Lesson Summary. During this lesson you identified the five steps of the logical plan used in solving word problems. You also translated English expressions into mathematical symbols. During the next lesson you will use the logical plan to solve word problems. If you had a difficult time with the language, review this section before moving on.

Lesson 2. SOLVING WORD PROBLEMS

LEARNING OBJECTIVE

Given word problems, apply the logical plan to solve each word problem.

6201. Word Problems

If we were to attack word problems with a plan just as we attack objectives by first organizing everything into a logical sequence as with operation orders, patrol orders, mission statements, commander's intent, and so on, much of the difficulty and fear these problems inspire would be replaced with confidence. Let's apply what you learned in previous lessons to solve word problems.

6202. Problems Involving Formulas

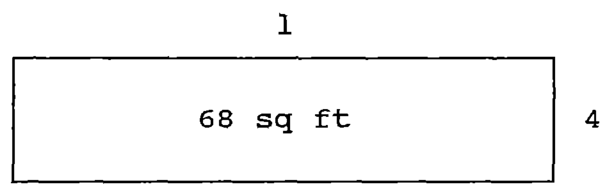
Problems involving formulas are by far the easiest type of word problems since the formula is the equation and it is necessary only to plug the pertinent items into it. The type of formula most often encountered will probably involve some aspect of geometric figures. Conversely, keep in mind that whenever the problem discusses a geometric figure, there is a formula which can aid you. Always sketch the figure, label the parts with the information that you have, and then apply the proper formula to the information.

<u>Example:</u>

(1) Read the problem.

The company storage space is 4 feet wide and has an area of 68 square feet. What is the length?

(2) Determine unknowns and represent them. Here a diagram does this.



(3) Write an equation.

$$A = lw$$
$$68 = l(4)$$

(4) Solve the equation.

$$A = lw$$
$$68 = l(4)$$
$$\frac{68}{4} = \frac{l(4)}{4}$$
$$17 = l$$

Check: $68 = 17(4)$
$68 = 68$

(5) Answer the problem.

The length of the company storage space is 17 feet.

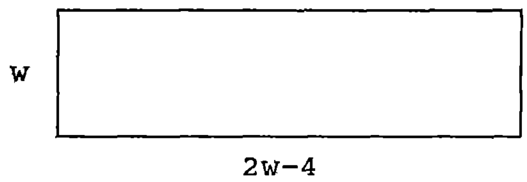
Example:

(1) Read the problem.

A picture of our National Ensign is 4 feet less in length than twice its width. To frame the picture, 76 feet of framing is needed. Find the dimensions of the picture.

(2) Determine the unknowns and represent them.

Draw the figure.



width = w
length = 2w-4
perimeter = 76

(3) Write an equation.

$$P = 2l + 2w$$
$$76 = 2(2w - 4) + 2(w)$$

(4) Solve the equation.

$$P = 2l + 2w$$
$$76 = 2(2w - 4) + 2(w)$$

Note: Do you recall the distributive property for multiplication? If not, you should review study unit 1.

$$76 = 4w - 8 + 2w$$
$$76 = 6w - 8$$
$$76 + 8 = 84 = 6w$$
$$84 = 6w$$
$$\frac{84}{6} = \frac{6w}{6}$$
$$14 = w$$

Example--cont'd.:

Note: Check your answer by inserting 14 for the variable w in the equation.

76 = 2(2w - 4) + 2(w)
76 = 2(2 x 14 - 4) + 2(14)
76 = 2(28 - 4) + 28
76 = 2(24) + 28
76 = 48 + 28
76 = 76

(5) Answer the problem.

The dimensions are: width equals 14 feet and the length equals 2w-4 or 24 feet. This can also be checked by substituting these values in the formula P = 2l + 2w.

P = 2l + 2w
76 = 2(24) + 2(14)
76 = 48 + 28
76 = 76

As you should notice, these five steps provide an easy systematic approach to problem solving (a logical plan). Let's take a look at another example.

Example:

(1) Read the problem.

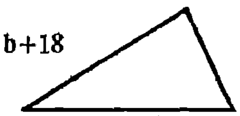
The area of the field that is used to do pullups and situps during the PFT is shaped like a triangle. The first side of the triangle is 18 feet more than the second side. The third side is 13 feet less than 3 times the second. It takes 130 feet of fencing to fence in the field. Find the length of each side of the field.

(2) Determine the unknowns and represent them.

Example--cont'd.:

Note: In reading the problem you should have noticed that the facts about the first and third sides are expressed as they relate to the second side. The second side, then, should be the first unknown that you represent.

Draw the figure.



Let b = second side

Now with this single letter variable as a start, you should be able to translate the language of the problem into symbols.

Let b + 18 = first side
Let 3b - 13 = 3d side

(3) Write an equation.

The formula is: $P = a + b + c$

$$130 = (b + 18) + b + (3b - 13)$$

Note: Parentheses are not needed but are inserted to show the three different sides.

(4) Solve the equation.

$$130 = (b + 18) + b + (3b - 13)$$
$$130 = b + 18 + b + 3b - 13$$
$$130 = 5b + 5$$
$$130 - 5 = 5b + 5 - 5$$
$$125 = 5b$$
$$\frac{125}{5} = \frac{5b}{5}$$
$$25 = b$$

Check:

$$130 = (b + 18) + b + (3b - 13)$$
$$130 = (25 + 18) + 25 + (3 \times 25 - 13)$$
$$130 = 43 + 25 + 75 - 13$$
$$130 = 130$$

Example--cont'd.:

(5) Answer the question.

To do this, we must go back to the unknown representation.

$$b = 25 \text{ (second side)}$$
$$b + 18 = 43 \text{ (first side)}$$
$$3b - 13 = 62 \text{ (third side)}$$

Check:

$$P = a + b + c$$
$$130 = 43 + 25 + 62$$
$$130 = 130$$

This time, let's take a problem that's a little more complicated. While applying the five steps, see if you can anticipate the steps of the solution before they are presented.

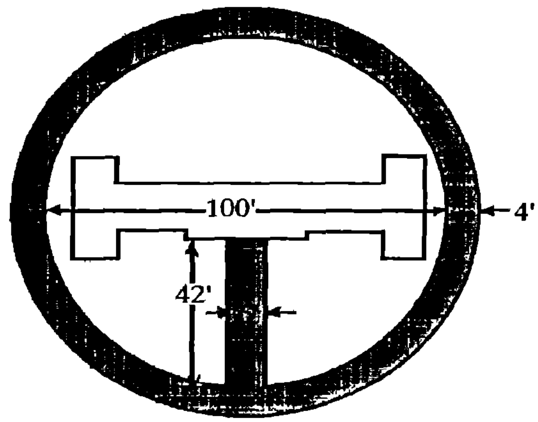
Example:

(1) Read the problem.

The combat engineer platoon received a mission to build a circular sidewalk around the division headquarters building. The walk will be 4 feet wide and 4 inches thick. In addition to the circular walk, there will be a 6 foot wide walk from the circular walk to the entrance. Sgt Readymix, the combat engineer chief, and his men are ready to go to work when the concrete arrives. He has ordered 19 1/2 cubic yards of concrete. Doublecheck Sgt Readymix to see if he has ordered the correct amount of concrete.

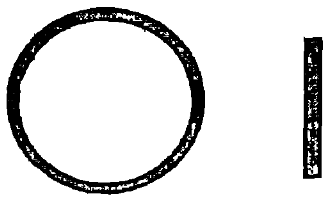
Example--cont'd.:

Note: Use the illustration provided for all dimensions.



(2) Determine the unknowns and represent them.

Here, all of the needed information is given in the illustration. The only real unknown is the answer. The biggest problem for you now is to select the formula or formulas that are to be used. What geometric forms are involved? If you said a cylinder and rectangular solid, you're correct! The illustration below shows them separated.

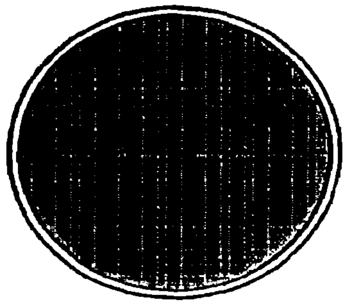


The sidewalk is to be 4 inches thick, so keep in mind that each figure is three dimensional. Several methods can be used to solve this problem. One method will be presented, but it is not necessarily the only method. Regardless of the method, the key to this problem is finding the area of the sidewalk. What formulas will you use to solve the problem? Let's take a look.

Example--cont'd.:

(3) Write an equation.

To find the area of the base of the circular sidewalk, use the formula $A = \pi r^2$. First, we must use it with the information from the large circle (entire area including the sidewalk) and then with the information from the shaded portion of the circle as illustrated.



If the large area is found and then the area of the shaded portion inside is subtracted from it, the remainder will be the area of the sidewalk. Let's write the equation.

Large area		Inside area
$A = \pi r^2$		$A = \pi r^2$
$A = 3.14(54^2)$	minus	$A = 3.14(50^2)$

Note: Keep in mind that 4 inches of thickness must be considered and converted to feet. The difference of the two areas multiplied by 1/3 foot (4 inches) will give you the volume in cubic feet of the circular walk. The short straight sidewalk is a rectangular solid. Do you recall the formula used to determine the volume of a rectangular solid? You're correct if you said V = lwh. Refer to the first illustration in this example for the dimensions.

Then write the equation for the rectangular solid and enter the information.

V = lwh
V = 42 x 6 x 1/3

(4) Solve the equations.

Example--cont'd.:

Large circle	Inside circle	Rectangular Solid
$A = \pi r^2$	$A = \pi r^2$	V = lwh
$A = 3.14(54^2)$	$A = 3.14(50^2)$	V = 42 x 6 x 1/3
A = 3.14(2916)	A = 3.14(2500)	V = 84 cu ft
A = 9156.24	A = 7850	

(5) Answer the problem.

In this case, the solutions to the formulas are not the answers to the problems. As indicated, we must find the difference between the two circle areas. The circular sidewalk is a cylinder. Its volume is the difference in areas times the thickness or height, in this case 1/3 foot.

Difference:

```
  9156.24 sq ft
- 7850.00 sq ft
  1306.24 sq ft
```

Multiply this answer by 1/3 foot

$$1306.24 \times \frac{1}{3} = 435.41 \text{ cu ft}$$

Add the two volumes

```
  435.41 cu ft (circular sidewalk)
+  84.00 cu ft (straight sidewalk)
  519.41 cu ft (concrete needed)
```

Is this the answer? You should have said no. The problem asked for cubic yards, not cubic feet. Since there are 27 cubic feet in 1 cubic yard (3 x 3 x 3), you must divide 519.41 by 27, and this answer will be the solution to the problem.

519.41 ÷ 27 = 19.23 cu yd

Check:

Looking back at the original problem, you'll see that Sgt Readymix ordered 19 1/2 cubic yards. This is correct for the job (the .23 cubic yards was rounded up to the nearest 1/2 cubic yard).

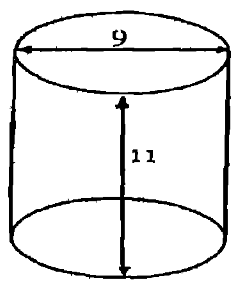
You have one more problem to solve. But, before you attack it, go ahead and take a break; you deserve it!

Welcome back! Your last problem involves a cylinder. Do you recall the formula to determine the volume of a cylinder? If you said $V = \pi r^2 h$, you're correct! Move on and solve your problem.

<u>Example</u>:

(1) Read the problem.

The water storage tank illustrated has a height of 11 meters and a diameter of 9 meters. What is the capacity, in gallons, of the tank if there are approximately 7 1/2 gallons in 1 cubic foot of water? Additionally, if the tank is hooked to a shower unit and each Marine is allowed 5 gallons of water, how many Marines can take a shower before the tank is empty ?



You should see that this problem involves several steps. As with your previous problem, this one can be solved in several ways, but only one will be shown. Have fun!

(2) Determine the unknowns and represent them.

In this problem the unknowns have been given. The radius is 9 meters and height is 11 meters.

<u>Note</u>: Before using the formula, what should you do about the dimensions in meters? You should have converted them to feet. It is a good practice to do your conversions (if any are necessary) at the beginning of your problem. In this case, you will change the meters to feet because you will eventually need to know cubic feet to determine gallons. One meter equals approximately 3.281 feet.

Example--cont'd.:

$$9 \text{ m} = 9 \times 3.281 = 29.529 \text{ ft}$$
$$11 \text{ m} = 11 \times 3.281 = 36.091 \text{ ft}$$

Round off to two places.

29.529 ft = 29.53 ft
36.091 ft = 36.09 ft

(3) Write an equation.

Using the formula for the volume of a cylinder, $V = \pi r^2 h$, plug in the variables.

$$V = \pi r^2 h$$
$$V = 3.14(14.77^2)36.09$$

Remember that the diameter is 29.53 feet and that the radius is 1/2 of the diameter. In this case the radius is 14.77 feet (rounded two places).

(4) Solve the equation.

$$V = \pi r^2 h$$
$$V = 3.14(14.77^2)36.09$$
$$V = 3.14(218.153)36.09$$
$$V = 24{,}721.65 \text{ cu ft}$$

This is the capacity of the tank. You now must determine how many Marines it will take to empty the tank.

If every cubic foot contains 7 1/2 gallons, then the product of 24,721.65 and 7 1/2 will be the gallon capacity of the tank.

24,721.65 x 7 1/2 = 185,412.38

If each man is allowed 5 gallons of water, then dividing the capacity of the tank by 5 will tell you how many men can take a shower before the tank is empty.

185,412.38 ÷ 5 = 37,082.47 = 37,082

(5) Answer the problem.

The capacity of the tank is 185,412.38 gallons of water. It will take 37,083 Marines to empty the tank.

By now you should have a pretty good understanding of the five steps of the logical plan used in solving word problems. If not, review this section before moving on.

6203. Miscellaneous Word Problems

Depending on your occupation, you may or may not have the opportunity or occasion to solve word problems other than those involving a formula or a simple arithmetic operation. However, being able to reason with numbers builds confidence in your own abilities and helps you to develop the logical and analytical thinking needed in many Marine Corps endeavors. To look at a problem, analyze it, and come up with an answer or decision is a desirable ability for Marines. The random problems in this paragraph may or may not apply to you and your job. The key thing is that they stimulate you to think and allow you to carry away principles that can be transferred to similar problems in other situations. In some examples, complete problems will not be presented; you will be asked to translate English expressions to mathematical symbols. Keep in mind that even if an equation is not required to solve a problem, you should still follow a plan. By the way, can you recall all five steps? You're right, they are: (1) read the problem, (2) determine the unknowns and represent them, (3) write an equation, (4) solve the equation, and (5) answer the question. Now let's have some fun by solving the next four problems. You may or may not decide to use the five step plan. If not, apply some common sense to decide whether your answers seem logical. Once you have completed the problems, compare your answers with the ones provided.

<u>Example</u>:

Shore party has 60 men working in 6 dumps on the beach. If none of the dumps have less than 7 Marines and no more than 18, what is the minimum number of Marines that can work in any two of these dumps?

This problem does not have a practical value; it is a number exercise. At first glance, the problem may seem complicated, but it is really quite basic. No equation is needed, just read it carefully. It asks for the minimum number of Marines for two of the dumps. If 7 is the minimum number of Marines for one dump, then 7 Marines x 2 dumps equals a total of 14 Marines. Just to be certain, we must check to be sure that this will not make any of the other dumps exceed 18. From the total of 60, subtract 14 which leaves 46 for the four other dumps, or 11 Marines for two of the dumps and 12 for the other two. This <u>is</u> within the limits set, so our answer is OK.

This next problem is a little more difficult to solve, so use the five step plan to solve it.

<u>Example</u>:

(1) Read the problem.

There are 40 questions on a promotion test. Each correct answer is worth one point and two points are deducted for each wrong answer. No points are added or deducted for questions not answered. If LCpl Claymore got a score of 15 and he had 5 questions wrong, what fractional part of the test did he answer?

(2) Determine the unknowns and represent them.

There are several ways to get the answer. The problem will fit very nicely into an equation. From the information given, we know that the score is equal to the number correct minus two times the number incorrect.

(3) Write an equation.

C = correct
I = incorrect
$C - 2I = 15$

We know that there were 5 incorrect problems. Plug this into the equation.

$$C - 2(5) = 15$$

(4) Solve the equation.

$$C - 2I = 15$$
$$C - 2(5) = 15$$
$$C - 10 = 15$$
$$C = 25$$

Check:

$$25 - 10 = 15$$
$$15 = 15$$

(5) Answer the problem.

Lcpl Claymore got 25 correct and 5 incorrect. This means he answered 30 of the 40 problems, or the fractional part 30/40 or 3/4.

Example:

A local discount store has vitamins on sale for 21 cents a dozen. The PX has the same item 100 for $1.50. How much is saved per dozen by buying the larger amount?

After reading the problem, it can be determined that algebra is not needed, just two simple arithmetic operations. First, the problem asks for a comparison of dozens. One price is already in dozens. The other can be converted several ways. Since 100 vitamins cost $1.50, and you remember from earlier in the course about dividing by powers of ten, you know that each vitamin costs 1.5 cents.

If one costs 1.5 cents, then 12 will cost 1.5 x 12 = 18 cents. The difference between the two prices is 21 cents minus 18 cents or 3 cents. So you save 3 cents per dozen by buying the larger amount.

Note: This was a very basic type of problem, but also a practical one that occurs often. Keep it in mind that solving this type of problem could save you money.

Example:

The indicator of a fuel tank shows 1/5 full. After the tanker delivers 165 gallons of fuel, the indicator shows 4/5 full. What is the capacity of the fuel tank?

The first thing to observe is that if the fuel tank was 1/5 full at the start and 4/5 full after the fuel was delivered, then the amount delivered was 4/5 minus 1/5 or 3/5 of the total capacity. We also know that this 3/5 is the same as 165 gallons. If the capacity of the fuel tank is x, then 165 gallons equals 3/5 of x.

$$165 \text{ gal} = \frac{3}{5}x$$

$$\frac{5}{3} \times \frac{165}{1} = \frac{825}{3} = 275$$

$$275 = x$$

Example--cont'd.:

Check:

$$165 = \frac{3}{5}(275)$$

$$165 = 165$$

The capacity of the tank is 275 gallons.

Note: Of the four problems provided, only one was solved using the five steps. This was done intentionally to show you that it isn't always necessary to use the five steps as long as you follow some kind of a plan.

Lesson Summary. In this lesson you were provided with the tools necessary to solve word problems by employing a logical plan and the language used to translate English expressions into mathematical symbols. If you had a difficult time with this area, review this section before moving on to the unit exercise.

Unit Exercise: Complete items 1 through 21 by performing the action required. Check your response against those listed at the end of this study unit.

1. Select the five steps to the logical plan.

   a. Read the problem, determine the unknowns and list them, write an equation, solve the equation, and answer the question.

   b. Read the problem, determine the unknowns and represent them, calculate, solve the equation, and answer the question.

   c. Read the problem, determine the unknowns and represent them, write a solution, solve the equation, and answer the question.

   d. Read the problem, determine the unknowns and represent them, write an equation, solve the equation, and answer the question.

Complete items 2 through 11 by translating the English expression into mathematical symbols.

2. The difference between 2x and five.

   a. 2x - 5　　c. 5 - 2x

   b. 2x + 5　　d. 5 x 2x

3. c added to d.

   a. c - d　　c. c + d
   b. d - c　　d. d x c

4. Fifteen decreased by n.

   a. 15 + n　　c. n - 15
   b. 15 - n　　d. n x 15

5. The product of 6 and a.

   a. 6 + a　　c. 6a
   b. 6 - a　　d. 6 ÷ a

6. Four more than 2 times eight.

   a. (2 + 8) x 4　　c. (4 x 2) + 8
   b. 2 x (8 + 4)　　d. 4 + (2 x 8)

7. The difference between twice x and half of y.

   a. 2x - y/2　　c. y/2 - 2x
   b. 2x + y/2　　d. y/2 + 2x

8. Six times the sum of n and 3.

   a. 6(n + 3)　　c. 6 + n(3)
   b. 6(n - 3)　　d. 6(n) + 3

9. Twenty-seven reduced by 5 times d.

   a. 27 - 5 + d　　c. (27 - 5)d
   b. 27 + d(5)　　d. 27 - 5d

10. Five divided by the sum of 4 times x and 3 times n.

   a. 5 ÷ 4x + 3 + n　　c. 5 ÷ 4 + x(3n)
   b. 5 ÷ 4(x3) x n　　d. 5 ÷ (4x + 3n)

11. One-half of the sum of r and s.

   a. rs ÷ 2　　c. r + s ÷ 2
   b. r ÷ s + 2　　d. r - s ÷ 2

Complete items 12 through 21 by performing the action required.

12. There are twelve rows of seats in the bleachers. Each row is n inches above the one in front. How high is the top row?

a. 12	c. 12 + n
b. 12n	d. 12 - n

13. One number is 4/5 of another number. If the larger number is represented by m, write an equation stating that the sum of the two numbers is 108.

a. 4/5m + m = 108	c. 4/5 + m(m) = 108
b. 4/5m - m = 108	d. 4/5 - m(m) = 108

14. Marines in the battalion were evaluated for two years on their rifle marksmanship scores. Out of the battalion, 400 Marines scored expert during the second year of training. If this is an increase of x over the first year, express the number who were experts at the beginning of the first year.

a. 400 ÷ x	c. 400 + x
b. 400x	d. 400 - x

15. At the PX, a notebook costs 28 cents more than a pencil. Represent the total cost of 5 pencils and 2 notebooks. Let x represent the cost on one pencil.

a. 5 + x 2(x + 28)	c. 5x - 2(x - 28)
b. 5 - x 2(x - 28)	d. 5x + 2(x + 28)

16. Several Marines pooled their resources and bought a used car. One of them owned 5/8 interest in it. He sold 1/5 of his interest to another Marine for $100. What was the original price of the car?

a. $900.00	c. $800.00
b. $850.00	d. $750.00

17. In 9 weeks, a moonlighting Marine working 5 nights a week made $454.50. How much did he make per day?

a. $10.10	c. $9.40
b. $9 .80	d. $9.10

18. Alpha Company plans to raise $500 for the Navy Relief drive by selling chances on a motorcycle. If expenses are $250, what is the minimum number of tickets at 75 cents each that must be sold to reach the goal?

a. 500
b. 650
c. 800
d. 1000

19. After several days of working out, 20 percent of the candidates for a football team were told to turn in their jerseys. This left 32 players on the team. How many tried out originally?

a. 30
b. 35
c. 40
d. 45

20. A badminton court 44 feet long is 4 feet longer than twice its width. Find the width.

a. 22 in.
b. 20 in.
c. 22 ft
d. 20 ft

21. The troop space in an LVT measures 5ft 6 in. high, 7 ft 3 in. wide, and 11 ft 5 in. long. If boat space for one Marine is considered to be 13.5 cu ft, how many Marines can be jammed into an LVT? If there are 5 tractors available, could a company of 195 men be carried in one wave?

a. 34 Marines, no, 170 men per 5 tractors
b. 34 Marines, yes, 195 men per 5 tractors
c. 35 Marines, no, 170 men per 5 tractors
d. 35 Marines, yes, 195 men per 5 tractors

UNIT SUMMARY

In this study unit you employed logical plans and the language used to translate English expressions into mathematical symbols to solve word problems.

Exercise Solutions

		Reference
1.	d. Read the problem, determine the unknowns and represent them, write an equation, solve the equation, and answer the question.	6102
2.	a. 2x - 5	6103
3.	c. c + d	6103
4.	b. 15 - n	6103
5.	c. 6a	6103
6.	d. 4 + (2 x 8)	6103
7.	a. 2x - y/2	6103

Reference

8.	a.	6(n + 3)	6103
9.	d.	27 - 5d	6103
10.	d.	5 ÷ (4x + 3n)	6103
11.	c.	r + s ÷ 2	6103
12.	b.	12n	6103
13.	a.	4/5m + m = 108	6103
14.	d.	400 - x	6103
15.	d.	5x + 2(x + 28)	6103
16.	c.	$800.00	6202

1/5 = 20%

$\frac{\$100.00}{x} = \frac{20\%}{100\%}$ (Let x equal 5/8 of car)

$\frac{100}{x} = \frac{.20}{1.00}$

.20x = 100

$\frac{.20x}{.20} = \frac{100}{.20}$

x = $500.00 (Total investment for 5/8 of car)

5/8 = 62.5%

$\frac{\$500}{y} = \frac{62.5\%}{100\%}$ (Let y represent the original price of the car)

$\frac{500}{y} = \frac{.625}{1.00}$

.625y = 500

$\frac{.625y}{.625} = \frac{500}{.625}$

y = $800.00 (Original price of the car)

17. a. $10.10 6202

5 nights a week for 9 week equals 45 nights

$454.50 ÷ 45 = $10.10 per night

18. d. 1,000 6202

$500.00 + $250.00 = $750.00

750 ÷ .75 = 1,000 tickets must be sold

Reference

19. c. Forty players tried out originally. 6202

$\frac{32}{x} = \frac{80\%}{100\%}$ (Let x = total players that tried out)

$\frac{32}{x} = \frac{.80}{1.00}$

$.8x = 32$

$\frac{.8x}{.8} = \frac{32}{.8}$

x = 40 (Players that originally tried out)

20. d. 20 ft 6202

$\frac{44 \text{ ft} - 4 \text{ ft}}{2} = 20 \text{ ft}$

21. a. 34 Marines, no, 170 men per 5 tractors 6202

5' 6" = 66" height
7' 3" = 87" width
11' 5" = 137" length

(Formula for volume of a rectangular solid is lwh)

```
  137"        11,919"
x  87"       x    66"
11,919"      786,654
```

(1728 cu in equals 1 cu ft)

786,654 ÷ 1728 = 455.23958 = 455.24 cu ft

(13.5 cu ft per Marine)

455.24 ÷ 13.5 = 33.72 = 34 Marines in one LVT

34 x 5 = 170 Marines that can be carried in five LVT's

APPENDIX A

FORMULAS

The formulas listed below are given to help you prepare for the examination. You may want to tear out this page and use it to study from.

1. $P = 4s$
2. $A = s^2$
3. $P = 2l + 2w$
4. $A = lw$
5. $P = 2b + 2w$
6. $A = bh$
7. $P = a + b + c + d$
8. $A = 1/2h(b_1 + b_2)$
9. $P = a + b + c$
10. $A = 1/2bh$
11. $C = 2\pi r$
12. $C = \pi d$
13. $A = \pi r^2$
14. $SA = 6(face^2)$
15. $V = f^3$
16. $SA = 2(h \times w) + 2(h \times l) + 2(w \times l)$
17. $V = lwh$
18. $LSA = 2\pi rh$
19. $TSA = 2\pi r(r + h)$
20. $V = \pi r^2h$
21. $a^2 + b^2 = c^2$

MATH FOR MARINES

REVIEW LESSON EXAMINATION

INSTRUCTIONS: This review lesson is designed to aid you in preparing for your final exam. You should try to complete this lesson using only Appendix A and your protractor. If you do not know an answer, look it up in the text and remember what it is. The enclosed answer sheet must be filled out according to the instructions on its reverse side and mailed to MCI using the envelope provided. The items you miss will be listed with references on a feedback sheet (MCI R-69) which will be mailed to your commanding officer with your final examination. You should study the reference material for the items you missed before taking the final examination.

Select the ONE answer which BEST completes the statement or answers the item. After the corresponding number on the answer sheet, blacken the appropriate circle.

1. What single factor made the Hindu-Arabic number system superior?

   a. Ability to perform addition
   b. Compounding of powers of ten
   c. Ability to repeat numbers
   d. Invention of zero

2. The two purposes of the digits in any number are to show the

   a. cardinal and place values.
   b. cardinal and face values.
   c. natural and place values.
   d. natural and face values.

3. Which set consists entirely of natural numbers?

   a. 5, 14, 17.8
   b. 3/4, 4, 12
   c. 0, 6, 9
   d. 4, 12, 31

4. Which example illustrates the principle of closure?

   a. 5 - 8 =
   b. 5 + 8 =
   c. 5 ÷ 8 =
   d. 8 ÷ 5 =

5. Which answer to the problems below is an integer?

   a. 5 ÷ 3
   b. 11.6 - 3
   c. 6 - 10
   d. 6 1/4 x 5

6. Rational numbers can be expressed as

a. real numbers.
b. decimals.
c. whole numbers.
d. fractions.

7. Which example illustrates the reflexive property of equality?

a. 28 = x
b. x = y
c. x = x
d. y = 28

8. When you write a statement such as a = 9, what axiom of equality allows you to change it to 9 = a?

a. Commutative property
b. Symmetric property
c. Reflexive property
d. Distributive property

9. What axiom of equality applies to this statement? If x = y and y = z, then x = z.

a. Transitive property
b. Commutative property
c. Reflexive property
d. Distributive property

10. The addition of 54 + 81 can be written 81 + 54 because of the ____________ property.

a. commutative
b. associative
c. reflexive
d. transitive

11. Which example illustrates the associative property for addition?

a. 3 + 7 + 8 = 18
b. 18 = 8 + 7 + 3
c. 8 + 7 + 3 = 9 + 9
d. (3 + 7) + 8 = (8 + 3) + 7

12. 6 x 8 = 8 x 6 illustrates the _____________ property for multiplication.

a. distributive
b. commutative
c. transitive
d. symmetric

13. Which example illustrates the associative property for multiplication?

a. 3 x 9 x 2 = 54
b. 54 = 3 x 9 x 2
c. 3 + 9 + 2 = 2 + 9 + 3
d. (2 x 9) x 3 = 2 x (3 x 9)

14. The law that states multiplication can be distributed over addition is the ____________________ property.

a. symmetric
b. transitive
c. commutative
d. distributive

15. Which example best describes the distributive property?

a. 6 x (4 + 3) = (6 x 4) + (6 x 3)
b. 6 x (4 + 3)
c. 24 x 18
d. 42

Solve items 16 through 17 using the zero and unity elements by performing the actions required.

16. 12 x 14 ÷ 0 =

a. Impossible
b. 168
c. 0
d. 1/64

17. 9 ÷ 1 =

a. 1
b. 1/9
c. 9
d. 0

Simplify items 18 through 19 using symbols of grouping by performing the actions required.

18. (10 ÷ 2) + 9

a. 29
b. 17
c. 14
d. 9

19. $10\ \overline{15 \times 2}$

a. 30
b. 152
c. 300
d. 315

Solve items 20 through 22 using the rules for order of operations by performing the action required.

20. (6 + 4 + 2) ÷ 2 + 2

a. 12
b. 9
c. 8
d. 3

21. 5[8 + 8] ÷ 2 + 5

a. 45
b. 43
c. 35
d. 33

22. (2 x 3 - 12 ÷ 3) ÷ 2

a. 4
b. 3
c. 2
d. 1

Complete items 23 through 24 by reducing each fraction to its lowest term.

23. $\frac{39}{51} = ?$

a. $\frac{15}{17}$
b. $\frac{13}{17}$
c. $\frac{15}{19}$
d. $\frac{13}{19}$

24. $\frac{340}{1260} = ?$

a. $\frac{17}{53}$
b. $\frac{19}{53}$
c. $\frac{17}{63}$
d. $\frac{19}{63}$

Complete items 25 through 26 by reducing each fraction to its higher term by supplying the missing numerator.

25. $\frac{3}{4} = \frac{?}{240}$

a. 110
b. 120
c. 180
d. 220

26. $\frac{9}{57} = \frac{?}{7011}$

a. 1105
b. 1107
c. 1109
d. 1112

Complete items 27 through 32 by performing the action required.

27.

$$\frac{15}{40} + \frac{5}{60} + \frac{4}{20} + \frac{3}{20}$$

a. $\frac{69}{120}$

b. $\frac{89}{120}$

c. $\frac{97}{120}$

d. $\frac{102}{120}$

28. $3\ \frac{6}{14} + 4\ \frac{22}{126} + 10\ \frac{4}{9}$

a. $16\ \frac{1}{19}$

b. $16\ \frac{1}{21}$

c. $18\ \frac{1}{19}$

d. $18\ \frac{1}{21}$

29.

$$7\ \frac{7}{36} - 5\ \frac{4}{9}$$

a. $3\ \frac{27}{36}$

b. $3\ \frac{1}{4}$

c. $1\ \frac{27}{36}$

d. $1\ \frac{1}{4}$

30. $10\ \frac{2}{3} \times 15$

a. 160

b. 165

c. 170

d. 175

31. $\frac{392}{448} \div \frac{168}{224}$

a. $1\ \frac{1}{6}$

b. $1\ \frac{1}{7}$

c. $1\ \frac{1}{8}$

d. $1\ \frac{1}{9}$

32. $114\ \frac{7}{12} \div 6\ \frac{7}{8}$

a. $14\ \frac{1}{3}$

b. $14\ \frac{2}{3}$

c. $16\ \frac{1}{3}$

d. $16\ \frac{2}{3}$

Complete items 33 through 34 by reducing the decimal fractions to common fractions in their lowest terms.

33. .268

a. $\frac{124}{500}$

b. $\frac{134}{500}$

c. $\frac{124}{750}$

d. $\frac{134}{750}$

34. .00475

a. $\frac{19}{4000}$

b. $\frac{19}{400}$

c. $\frac{19}{8000}$

d. $\frac{19}{800}$

Complete items 35 through 36 by reducing each common fraction to a decimal fraction (power of ten).

35. $\frac{15}{50}$

a. .13

b. .16

c. .20

d. .30

36. $\frac{47}{125}$

a. .976

b. .592

c. .492

d. .376

Complete items 37 through 40 by performing the action required.

37.
```
     6.38
     4.975
     3.1
  + 16.42
```

a. 29.875

b. 29.885

c. 30.785

d. 30.875

38.
```
    56.37
  - 18.48
```

a. 35.98

b. 35.89

c. 37.89

d. 37.98

39. $\begin{array}{r} 56.37 \\ \underline{\times\ \ 8.48} \end{array}$

a. 478.0176
b. 478.176
c. 578.0176
d. 578.176

40. $76.3\,\overline{)\,97.4200}$ Round to nearest hundredths (.00).

a. 1.28
b. 1.27
c. 1.38
d. 1.37

Complete items 41 through 42 by changing each item to a percent.

41. .22

a. .22%
b. 2.2%
c. 22%
d. 220%

42. $\frac{7}{40}$

a. .175%
b. 1.75%
c. 17.5%
d. 175%

Complete items 43 through 44 by changing from percents to fractions (reduced to lowest terms).

43. 60%

a. $\frac{3}{6}$
b. $\frac{3}{5}$
c. $\frac{3}{4}$
d. $\frac{2}{3}$

44. 87.5%

a. $\frac{4}{5}$
b. $\frac{5}{6}$
c. $\frac{6}{7}$
d. $\frac{7}{8}$

Complete items 45 through 51 by performing the action required.

45. During an equipment inventory, Cpl Pyle found 18 E-Tools missing from the list of 80. What is his percent of missing E-Tools?

a. 2.25%
b. 2.52%
c. 22.5%
d. 25.2%

46. Of 60 Marines in the platoon, 80% are qualified swimmers. How many Marines are qualified swimmers?

a. 42
b. 44
c. 46
d. 48

47. During the month of April (30 days), our company spent 28% of its time aboard ship. How many days did the company spend aboard ship? Round to the nearest day.

a. 5
b. 6
c. 7
d. 8

48. Ten percent of the platoon are attached to Company A. If there are 6 Marines attached to the Company A, how many Marines are in the platoon?

a. 60
b. 58
c. 50
d. 44

49. Which algebraic expression is a monomial term?

a. $6x + y + x + y$
b. $(xz)^4 - 1x^3$
c. $22a - 53a$
d. $56x^3y$

50. Which algebraic expression contains like terms?

a. $145abc^2$, $35abc$
b. xyz^5, $xy(z)^4$
c. $6a^2$, $(6a)^2$
d. $12xy$, $4xy$

51. Which algebraic expression is a polynomial term?

a. $4x - x + 7z^3$
b. $\frac{47xy}{2x - y}$
c. $\frac{7x + z}{xz}$
d. $64abc$

Complete items 52 through 56 by performing the proper operations of evaluation.

52. $a^3 - 2a^2 + a + 4$, $(a = 6)$

a. 584
b. 534
c. 154
d. 145

53. $\frac{x^2 + z^2}{y^2}$, (x = 15, y = 6, z = 9)

a. 4
b. 7
c. 3 1/2
d. 8 1/2

54. How many pounds of explosives will you need to cut fourteen 36" diameter trees? Round to the nearest pound using the algebraic expression: $P = \frac{D^2}{250}$

P = Pounds of explosives
D = Diameter of tree in inches
250 = constant

a. 70 lbs
b. 71 lbs
c. 72 lbs
d. 73 lbs

55. The objective is located 84 1/4 miles (distance) away, and you are moving at a <u>rate</u> of 5 mph. How long will it take you to arrive at the objective?
Use the algebraic expression: $T = \frac{D}{R}$

T = Time
D = Distance
R = Rate

a. 16.85 hours
b. 16.58 hours
c. 15.85 hours
d. 15.58 hours

56. You are in a defensive position with a front of 156 meters. Your platoon is responsible for the tactical wire emplacement of three belts. What is the total length of tactical wire (TAC(W)) required? Use the algebraic expression:

TAC(W) = 1.25 x length of front(LOF) x number of belts (# Belts).

a. 585 meters
b. 558 meters
c. 195 meters
d. 159 meters

Complete item 57 by determining if the equation is true or false.

57. 5 x 3 1/2 = 3 x 5 1/2

a. 16.5 = 17.5 (false)
b. 17.5 = 16.5 (false)
c. 17.5 = 17.5 (true)
d. 16.5 = 16.5 (true)

Complete item 58 by performing the action required and selecting the correct root.

58. $9h + 12.63 = 69.78$

a. $x = 7.53$
b. $x = 7.35$
c. $x = 6.53$
d. $x = 6.35$

Complete items 59 through 60 by determining whether these inequalities are true or false.

59. $5(9 - 5) - \frac{8 + 2}{5} \neq 4 + 8 \cdot 2$

a. $38 \neq 18$ (true)
b. $28 \neq 18$ (true)
c. $20 \neq 18$ (true)
d. $18 \neq 20$ (true)

60. $6 \cdot \frac{12 + 8}{5} > 6 \cdot 2$

a. $24 > 12$ (true)
b. $24 < 48$ (true)
c. $48 > 12$ (true)
d. $48 < 18$ (true)

Complete items 61 through 148 by performing the action required.

61. The combat engineers reported that the mountain road has a slope of 5% (5 feet of rise for every 100 feet of length). How much will the road rise in 1 mile?

a. 264 feet per mile
b. 246 feet per mile
c. 26.4 feet per mile
d. 24.6 feet per mile

62. You are in a defensive position and have been given 28 40mm rounds (M203 grenade launcher) for every 350 feet of front. How many rounds will you receive for every 450 feet of front?

a. 36
b. 34
c. 32
d. 30

63. One hundred and twenty-five Marines crossed the stream in 30 minutes. How many Marines will cross the stream in 60 minutes?

a. 205 | c. 250
b. 240 | d. 255

64. Three Marines out of 10 were wounded while on patrol. What percent was wounded?

a. 30% | c. 30.3%
b. 33% | d. 33.3%

65. Twelve of the 64 rifles inspected had unserviceable stocks. What percent of the rifles had unserviceable stocks?

a. 18% | c. 18.75%
b. 19% | d. 19.75%

66. The first leg on the land navigation course was 69,000 centimeters long. How many meters long was the first leg?

a. 6,900 | c. 69.0
b. 690 | d. 6.90

67. The enemy is located 28 kilometers west of our position. How many meters away is the enemy?

a. 2.8 | c. 2,800
b. 280 | d. 28,000

68. On a map with a scale of 1:50,000, the distance between point A and B is 49 centimeters. What is the actual ground distance in meters?

a. 2,450 | c. 245,000
b. 24,500 | d. 2,450,000

69. On a 1:25,000 map, 22.7 centimeters equals how many kilometers on the ground?

a. 5.675 | c. 56.75
b. 5.765 | d. 56.65

70. The platoon drank 216 liters of water. How many milliliters of water did the platoon drink?

a. 2,160 | c. 216,000
b. 21,600 | d. 2,160,000

71. The fuel bladder has 1237 liters of diesel fuel remaining. How many hectoliters of diesel fuel remain?

a. 1.237 | c. 123.7
b. 12.37 | d. 1237.0

72. Of 6.5 hectoliters of fuel, the private spilled 380 milliliters. How many liters of fuel did the private spill?

a. .38 c. 38
b. 3.8 d. 380

73. 570 milliliters of oil were poured from the 8.75 liter container. How many liters of oil remain?

a. 818 c. 8.18
b. 81.8 d. .818

74. A 6,540 kilogram pallet of ammo equals how many tonne?

a. 65.4 c. .654
b. 6.54 d. .0654

75. If the ship weighed 980 tonnes, how many kilograms does the ship weigh?

a. 9,800 c. 980,000
b. 98,000 d. 9,800,000

76. You just removed 60,000 grams of 5.56 ammo from the stock-pile of 176 kilograms. How many kilograms of ammo remain?

a. 1160 c. 11.6
b. 116 d. 1.16

77. You have 47 tonne plus 560,000 grams of C-4 explosives. How many kilograms of explosives do you have?

a. 47.56 c. 4,756
b. 475.6 d. 47,560

78. There are 4 boards measuring 4 feet 3 inches, 25 inches, 3 1/2 yards, and 4 3/4 feet. What is the total length of all 4 boards?

a. 20 ft 7 in c. 21 ft 7 in
b. 20 ft 9 in d. 22 ft 9 in

79. Each Marine in the patrol has a piece of fiber rope 6 yards 3 feet 17 inches long. There are 19 Marines in the patrol. What is the total length of all 19 segments of ropes?

a. 145 yd 5 ft 1 in
b. 144 yd 2 ft 11 in
c. 143 yd 5 ft 1 in
d. 141 yd 2 ft 11 in

80. The first container had 4 gallons 4 quarts 2 pint of diesel fuel, the second container had 3 gallons 2 quarts of diesel fuel, and the third container had 5 gallons 2 quarts 1 pint of diesel fuel. What was the total quantity of diesel fuel?

a. 14 gal 1 qt 1 pt
b. 14 gal 3 qt 1 pt
c. 15 gal 2 qt 1 pt
d. 15 gal 3 qt 1 pt

81. Each heater stove at the Mountain Warfare Training Camp burns 2 gallons 2 quarts of diesel fuel per day. There are 417 stoves in the camp. How much diesel fuel is burned daily?

a. 1,024 gal 2 qt
b. 1,042 gal 2 qt
c. 1,124 gal 2 qt
d. 1,142 gal 2 qt

82. You have 3 pallets loaded with supplies. One pallet weighs 1 ton, 400 pounds 12 ounces another pallet weighs 7 ton, 24 pounds 7 ounces and the last pallet weighs 1,950 lb. 14 oz. What is the total weight of the 3 pallets?

a. 8 ton 376 lb 1 oz
b. 8 ton 367 lb 1 oz
c. 9 ton 376 lb 1 oz
d. 9 ton 367 lb 1 oz

83. The utilities platoon crated 5 reciprocating water pumps weighing 490 pounds 10 ounces. What is the weight of one pump?

a. 97 lb 2 oz
b. 97 lb 4 oz
c. 98 lb 2 oz
d. 98 lb 4 oz

84. The distance that female Marines run for the PFT (physical fitness test) is 1 1/2 miles. How many kilometers do the female Marines run?

a. 1.2
b. 2.1
c. 2.4
d. 4.2

85. One of the combat conditioning courses is 5000 meters long. How many yards long is the combat conditioning course?

a. 51,555.56
b. 55,555.56
c. 5,155.56
d. 5,555.56

86. The enemy is located 56 miles east of our position. How many kilometers away is the enemy?

a. 89.9
b. 89.6
c. 86.9
d. 86.6

87. Pvt. Demolition found a landmine that is 10 inches in diameter. How many centimeters in diameter is it?

a. 25
b. 24
c. 23
d. 22

88. It took 574 milliliters of boiling water to purify the canteen cup. How many fluid ounces were needed to purify the cup?

a. 13.19
b. 13.91
c. 19.13
d. 19.31

89. The fuel can holds 4 liters of diesel. How many quarts does it hold?

a. 4.12
b. 4.21
c. 41.2
d. 42.1

90. It took 5.4 pints of chlorine to purify the water in the holding tank. How many liters of chlorine were needed?

a. 2.538
b. 2.583
c. 25.38
d. 25.83

91. It took 7.45 quarts of paint to touch up the 5 ton truck. How many liters of paint did it take?

a. 7.5770
b. 7.0775
c. 75.770
d. 70.775

92. The field jacket liner weighed 2,128 grams. How many ounces did the liner weigh?

a. 87
b. 86
c. 77
d. 76

93. The M203 grenade launcher weighs 1.35 kilograms. How many pounds does it weigh?

a. 3 lb.
b. 3.5 lb.
c. 4 lb.
d. 4.5 lb.

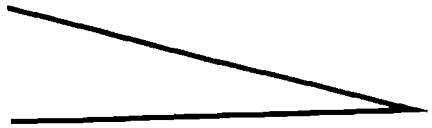
94. The combat engineers used 3,500 pounds of C-4 to breach the minefield. How many kilograms of explosives did they use?

a. 1475
b. 1457
c. 1575
d. 1755

95. The MK19 40mm machinegun tripod with gun cradle and traversing mechanism weighs 65 pounds. How many kilograms does it weigh?

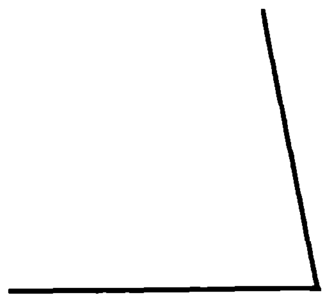
a. 26.25
b. 27.25
c. 28.25
d. 29.25

96. Using your protractor, find the measurement of the angle in degrees.



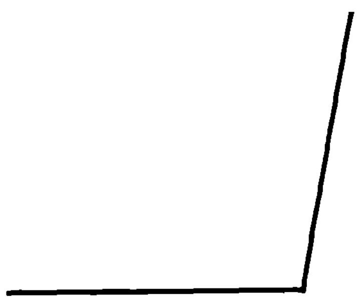
a. 16°
b. 18°
c. 20°
d. 22°

97. Using your protractor, find the measurement of the angle in degrees.



a. 72°
b. 75°
c. 79°
d. 82°

98. Using your protractor, find the measurement of the angle in degrees.

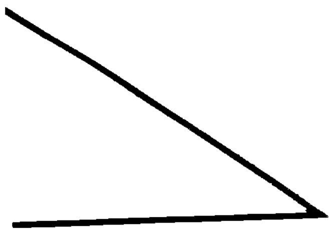


a. 94°
b. 99°
c. 101°
d. 104°

99. Using your protractor, find the measurement of the angle in degrees.

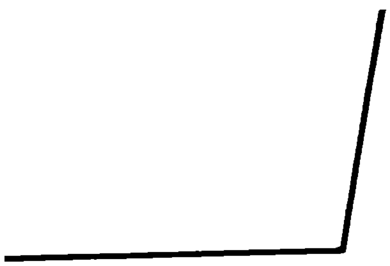
a. 135°
b. 139°
c. 142°
d. 146°

100. Measure and classify the angle shown.



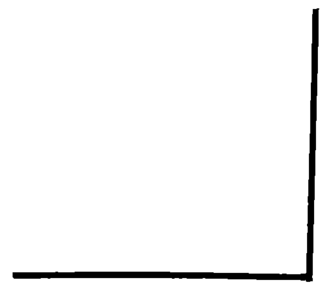
a. Right Angle
b. Obtuse Angle
c. Acute Angle
d. Straight Angle

101. Measure and classify the angle shown.



a. Right Angle
b. Obtuse Angle
c. Acute Angle
d. Straight Angle

102. Measure and classify the angle shown.



a. Right Angle
b. Obtuse Angle
c. Acute Angle
d. Straight Angle

103. $\begin{array}{r} 38^{\circ}\ 33' \\ \underline{+\ 166^{\circ}\ 26'} \end{array}$

a. $205^{\circ}\ 59'$
b. $205^{\circ}\ 49'$
c. $204^{\circ}\ 59'$
d. $204^{\circ}\ 49'$

104. $\begin{array}{r} 274^{\circ}\ 34' \\ \underline{-\ 180^{\circ}\ 54'} \end{array}$

a. $94^{\circ}\ 50'$
b. $94^{\circ}\ 40'$
c. $93^{\circ}\ 50'$
d. $93^{\circ}\ 40'$

105. $\begin{array}{r} 118^{\circ}\ 37'\ 39'' \\ \underline{+\ 140^{\circ}\ 42'\ 34''} \end{array}$

a. $249^{\circ}\ 20'\ 13''$
b. $249^{\circ}\ 11'\ 53''$
c. $259^{\circ}\ 20'\ 13''$
d. $259^{\circ}\ 11'\ 53''$

106. $\begin{array}{r} 124^{\circ}\ 12'\ 32'' \\ \underline{-\ 89^{\circ}\ 18'\ 54''} \end{array}$

a. $34^{\circ}\ 53'\ 38''$
b. $34^{\circ}\ 43'\ 28''$
c. $35^{\circ}\ 53'\ 38''$
d. $35^{\circ}\ 43'\ 28''$

107. The platform the sentry stood on was in the shape of a square. What is the perimeter of the platform if one side measures 60 inches?

a. 3,600 sq in
b. 3,660 sq in
c. 240 in
d. 420 in

108. The platform the sentry stood on was in the shape of a square. What is the area of the platform if one side measures 60 inches?

a. 3,660 sq in
b. 3,600 sq in
c. 420 in
d. 240 in

109. A broken window in the barracks is square shaped and measures 18 inches on one side. What is the perimeter of the broken window?

a. 342 sq in
b. 324 sq in
c. 74 in
d. 72 in

110. Find the area of the glass covering the top of the display case. The case is square and measures 18 inches on one side.

a. 324 sq in
b. 342 sq in
c. 72 in
d. 74 in

111. The cardboard used to display the Commandant's message was rectangular with a length of 26 inches and a width of 18 inches. What is the perimeter of the cardboard?

a. 486 sq in
b. 468 sq in
c. 88 in
d. 86 in

112. The pressure board found on top of the foreign antitank landmine was rectangular with a length of 14 inches and a width of 9 inches. How much area does the pressure board cover?

a. 162 sq in
b. 126 sq in
c. 64 in
d. 46 in

113. The opening in the bunker is rectangular with a length of 60 inches and a width of 48 inches. What is the perimeter of the opening?

a. 2,880 sq in
b. 2,680 sq in
c. 261 in
d. 216 in

114. A piece of plywood used to cover a hole in the bulkhead is rectangular with a length of 72 inches and a width of 39 inches. How much area does the piece of plywood cover?

a. 2,806 sq in
b. 2,808 sq in
c. 222 in
d. 224 in

115. The staging area used for the explosives was in the shape of a parallelogram with a width of 14 feet and a base of 36 feet. What was the perimeter of the staging area?

a. 298 sq ft
b. 288 sq ft
c. 100 ft
d. 50 ft

116. The sign used to identify the foreign armed forces was in the shape of a parallelogram with a base of 16 inches and a height of 10 inches. How much area did the sign cover?

a. 180 sq in
b. 160 sq in
c. 52 in
d. 54 in

117. The sandtable used during the training exercise covered an area shaped like a parallelogram with a width of 17.7 feet and a base of 54.3 feet. What was the perimeter of the sandtable?

a. 961.11 sq ft
b. 951.11 sq ft
c. 144 ft
d. 142 ft

118. A triple standard concertina fence used for temporary holding of prisoners of war covered an area shaped like a parallelogram with a width of 25 meters and a base of 72.7 meters. How much area did the fence cover?

a. 1817.5 sq m
b. 1871.5 sq m
c. 194.5 m
d. 195.4 m

119. What is the perimeter of the concrete pad that has the shape of a trapezoid measuring 21 feet by 9 feet by 28 feet by 9 feet.

a. 147 sq ft
b. 149 sq ft
c. 67 ft
d. 76 ft

120. The overlay on the map showed that the enemy had control over terrain that covered an area shaped like a trapezoid. The measurements listed on the map were given as: base 1 (b1) 10 miles, base 2 (b2) 18 miles and height (h) 8 miles. How much area does the enemy cover?

a. 112 sq mi
b. 82 sq mi
c. 44 mi
d. 43 mi

121. The PT field is shaped like a trapezoid measuring 700 feet by 1100 feet by 700 feet by 1200 feet. What is the perimeter of the PT field?

a. 421,100 sq ft
b. 412,100 sq ft
c. 3700 ft
d. 3600 ft

122. The tank park shaped like a trapezoid has a base 1 (b1) of 1100 feet, a base 2 (b2) of 1200 feet, and a height (h) of 700 feet. How much area does the tank park cover?

a. 850,000 sq ft
b. 805,000 sq ft
c. 3600 ft
d. 3700 ft

123. The standard NATO marker used to mark a biologically contaminated area is shaped like a triangle. It has the red letters BIO on a blue background and measures 20 centimeters by 20 centimeters by 28 centimeters. What is the perimeter of the standard NATO marker?

a. 208 sq cm	c. 86 cm
b. 280 sq cm	d. 68 cm

124. The biologically contaminated area was shown on the map to be covering terrain in the shape of a triangle. The two measurements listed on the map gave the dimensions of a base of 24 miles and a height of 17 miles. How much area shown on the map was biologically contaminated?

a. 204 sq mi	c. 41 mi
b. 240 sq mi	d. 44 mi

125. The security force formed in the shape of a triangle measuring 30 feet by 70 feet by 30 feet. What is the perimeter of the security force?

a. 3200 sq ft	c. 140 ft
b. 3300 sq ft	d. 130 ft

126. The enemy triangle wall was blocking the Marines field of fire. The base of the wall measured 70 feet and the height of the wall measured 28 feet. What area formed by the wall was blocking the Marines field of fire?

a. 980 sq ft	c. 98 ft
b. 890 sq ft	d. 89 ft

127. The enemy landmine that had a radius of 8.5 inches. What was its circumference?

a. 53.38 in	c. 49.38 in
b. 51.38 in	d. 47.38 in

128. A fuel can has a diameter of 15 inches. What is its circumference?

a. 48.1 in	c. 47.1 in
b. 48.6 in	d. 47.6 in

129. How much area will two trip flares illuminate if the illumination radius is 400m?

a. 540,400 sq m	c. 520,656 sq m
b. 524,400 sq m	d. 502,656 sq m

130. The enemy minefield was circular with a diameter of 600m. How much area did the enemy minefield cover?

a. 1,310,400 sq m
b. 1,130,400 sq m
c. 282,600 sq m
d. 228,600 sq m

131. The radius of a chemically contaminated area is 375m. How much area is chemically contaminated?

a. 441,562.75 sq m
b. 441,562.5 sq m
c. 441,526.75 sq m
d. 441,526.5 sq m

132. Find the surface area of a company supply box that is 6 feet wide, 6 feet high, and 6 feet long.

a. 261 sq ft
b. 216 sq ft
c. 261 cu ft
d. 216 cu ft

133. Find the volume of a company supply box that measures 6 feet wide, 6 feet high, and 6 feet long.

a. 261 sq ft
b. 216 sq ft
c. 261 cu ft
d. 216 cu ft

134. What is the surface area of a gas chamber that measures 13 feet wide, 13 feet high, and 13 feet long?

a. 1,014 sq ft
b. 1,041 sq ft
c. 2,179 cu ft
d. 2,197 cu ft

135. Find the volume of a tool room that measures 13 feet wide, 13 feet high, and 13 feet long.

a. 1,014 sq ft
b. 1,041 sq ft
c. 2,179 cu ft
d. 2,197 cu ft

136. Find the surface area of a milvan that measures 7 feet wide, 7 feet high, and 11 feet long.

a. 304 sq ft
b. 406 sq ft
c. 539 cu ft
d. 593 cu ft

137. What is the volume of a concrete obstacle that measures 7 feet wide, 7 feet high, and 11 feet long?

a. 304 sq ft
b. 406 sq ft
c. 539 cu ft
d. 593 cu ft

138. What is the surface area of a concrete bunker that measures 8 feet high, 8 feet wide, and 12 feet long?

a. 512 sq ft
b. 521 sq ft
c. 768 cu ft
d. 786 cu ft

139. What is the volume of the company storage room that measures 8 feet high, 8 feet wide, and 12 feet long?

a. 512 sq ft	c. 768 cu ft
b. 521 sq ft	d. 786 cu ft

140. Find the volume of a water tank that has a radius of 7.5 feet and a height of 30 feet.

a. 5,298.75 cu ft	c. 52,987.5 cu ft
b. 5,289.75 cu ft	d. 52,897.5 cu ft

141. Find the volume of an anti-tank landmine that has a radius of 5.5 inches and a height of 8 inches.

a. 87.5 cu in	c. 795.88 cu in
b. 78.5 cu in	d. 759.88 cu in

142. What is the square root of 2,076?

a. 54.56	c. 45.56
b. 54.65	d. 45.65

143. What is the square root of 1,685,225?

a. 1289.16	c. 12891.62
b. 1298.16	d. 12981.62

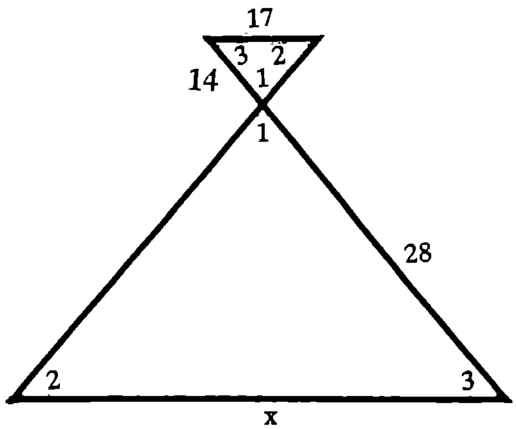
144. Find the missing side (c) of a right triangle that has a side (a) of 7 inches and a side (b) of 8 inches.

a. 10.63 in	c. 9.63 in
b. 10.36 in	d. 9.36 in

145. Find the missing side (c) of a right triangle that has a side (a) of 3 inches and a side (b) of 9 inches.

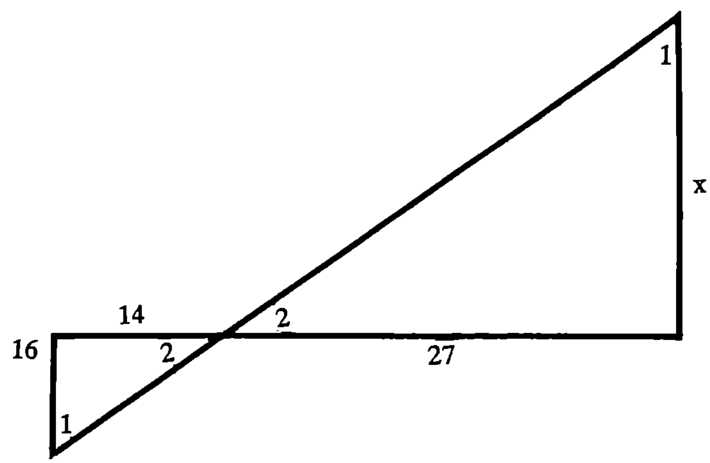
a. 9.48 in	c. 9.68 in
b. 9.58 in	d. 9.85 in

146. Find the missing side (x) of the triangle illustrated.



a. 30
b. 32
c. 34
d. 36

147. Find the missing side (x) of the triangle illustrated.



a. 30.58
b. 30.85
c. 31.58
d. 31.85

148. Select the five steps to the logical plan.

   a. Read the problem, determine the knowns and list them, write an equation, solve the equation, and answer the question.

   b. Read the problem, determine the unknowns and represent them, calculate, solve the equation, and answer the question.

   c. Read the problem, determine the unknowns and represent them, write a solution, solve the equation, and answer the question.

   d. Read the problem, determine the unknowns and represent them, write an equation, solve the equation, and answer the question.

Complete items 149 through 152 by translating the English expression into mathematical symbols.

149. The product of 9 and a.

   a. 9 + a
   b. 9 - a
   c. 9a
   d. a9(9)

150. Eight more than seven times nine.

   a. 7 + 9 x 8
   b. 7 x 9 + 8
   c. 8 x 7 + 9
   d. 8 + 7 x 9

151. Thirty-one reduced by three times d.

   a. 31 ÷ 3 + d
   b. 31 x d(3)
   c. 31 + 3d
   d. 31 - 3d

152. Twenty-two divided by the sum of eight times x and two times n.

   a. 22 ÷ 8x + 2 + n
   b. 22 ÷ 8(x2) x n
   c. 22 ÷ 8 + x(2n)
   d. 22 ÷ (8x + 2n)

Complete items 153 through 160 by performing the action required.

153. One number is seven-eights of another number. If the larger number is represented by m, write an equation stating that the sum of the two numbers is 96.

   a. 7/8m + m = 96
   b. 7/8m - m = 96
   c. 7/8 + m(m) = 96
   d. 7/8 - m(m) = 96

154. Marines in the battalion were evaluated for two years in a row on their rifle scores at the rifle range. Three-hundred Marines scored expert during the second year of training. If this is an increase of x over the first year, which expressions represent the number who were expert at the begining of the first year?

a. 300 ÷ x
b. 300x
c. 300 + x
d. 300 - x

155. At the PX, a notebook costs 88 cents more than a pencil. Represent the total cost of 9 pencils and 3 notebooks. Let x represent the cost on one pencil.

a. 9 + x 3(x + 88)
b. 9 - x 3(x - 88)
c. 9x - 3(x - 88)
d. 9x + 3(x + 88)

156. Several Marines pooled their resources and bought a used car. One of them owned 5/8 interest in it. He sold 1/5 of his interest to another Marine for $300. What was the original price of the car?

a. $2,400.00
b. $2,350.00
c. $2,200.00
d. $2,100.00

157. Alpha Company plans to raise $450 for the Navy Relief drive by selling chances on a motorcycle. If expenses are $150, what is the minimum number of tickets at 75 cents each that must be sold to reach the goal?

a. 500
b. 650
c. 800
d. 1000

158. After several days of working out, 10% of the Marines on a football team were told to turn in their jerseys. This left 36 Marines on the team. How many Marines tried out originally?

a. 30
b. 35
c. 40
d. 45

159. The inspection area the Marines formed in is 66 feet long, which is 6 feet longer than twice its width. What is the width of the inspection area?

a. 32 in
b. 30 in
c. 32 ft
d. 30 ft

160. A plane flew a triangular route for a total distance of 149 miles. If the longest leg of the flight was twice the shortest leg, and the third leg was one mile less than the longest leg, what was the length of the longest leg of the flight?

a. 30 miles
b. 59 miles
c. 60 miles
d. 118 miles

LAST PAGE OF THE REVIEW LESSON

Review Lessons Solutions

		Reference
1.	d	1101c
2.	a	1101c
3.	d	1201
4.	b	1202a
5.	c	1301a
6.	d	1301b
7.	c	1401a
8.	b	1401b
9.	a	1401c
10.	a	1402a
11.	d	1402b
12.	b	1402c
13.	d	1402d
14.	d	1403
15.	a	1403
16.	a	1501a
17.	c	1501b
18.	c	1601
19.	c	1601
20.	d	1701abc
21.	a	1701abc
22.	c	1701abc
23.	b	2101
24.	c	2101
25.	c	2101
26.	b	2201
27.	b	2201
28.	d	2201
29.	d	2202
30.	a	2203
31.	b	2204
32.	d	2304
33.	b	2302
34.	a	2302
35.	d	2302
36.	d	2302
37.	d	2302
38.	c	2303
39.	a	2303
40.	a	2303
41.	c	2401
42.	c	2401
43.	b	2401

		Reference
44.	d	2401
45.	c	2401
46.	d	2401
47.	d	2401
48.	a	2401
49.	d	3102
50.	d	3103
51.	a	3104
52.	c	3105
53.	d	3105
54.	d	3105
55.	a	3105
56.	a	3105
57.	b	3201
58.	d	3202
59.	d	3202
60.	a	3202
61.	a	3302
62.	a	3302
63.	c	3302
64.	a	3302
65.	c	3303
66.	b	4102
67.	d	4102
68.	b	4102
69.	a	4102
70.	c	4103
71.	b	4103
72.	a	4103
73.	c	4103
74.	b	4104
75.	c	4104
76.	b	4104
77.	d	4104
78.	c	4202
79.	d	4202
80.	a	4203
81.	b	4203
82.	c	4204
83.	c	4204
84.	c	4301
85.	d	4301
86.	b	4301

		Reference
87.	a	4301
88.	c	4302
89.	b	4302
90.	a	4302
91.	a	4302
92.	d	4303
93.	a	4303
94.	c	4303
95.	d	4303
96.	a	5101
97.	c	5101
98.	b	5101
99.	d	5101
100.	c	5102
101.	b	5102
102.	a	5102
103.	c	5103
104.	d	5103
105.	c	5103
106.	a	5103
107.	c	5201
108.	b	5201
109.	d	5201
110.	a	5201
111.	c	5202
112.	b	5202
113.	d	5202
114.	b	5202
115.	c	5203
116.	b	5203
117.	c	5203
118.	a	5203
119.	c	5204
120	a	5204
121	c	5204
122.	b	5204
123.	d	5205
124.	a	5205
125.	d	5205
126.	a	5205
127.	a	5206
128.	b	5206
129.	d	5206

		Reference
130.	c	5207
131.	b	5207
132.	b	5301
133.	d	5301
134.	a	5301
135.	d	5301
136.	b	5301
137.	c	5301
138.	a	5301
139.	c	5301
140.	a	5301
141.	d	5301
142.	c	5302
143.	b	5302
144.	a	5302
145.	a	5302
146.	c	5302
147.	b	5302
148.	d	6102
149.	c	6103
150.	d	6103
151.	d	6103
152.	d	6103
153.	a	6104
154.	d	6104
155.	d	6104
156.	a	6104
157.	c	6104
158.	c	6104
159.	d	6104
160.	c	6104

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Marine Corps Engineer School, Math Work Book (1991)

NAVEDTRA 10069. Mathematics Volume 1 (1985)

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MCIO P1500.44c, Battle Skills Training/Essential Subjects Handbook (September 1989)

FM 5-25 Explosives and Demolitions

FM 5-34 Engineer Field Data

FM 5-35 Engineer's Reference Logistical Data

FM 20-32 Mine/Countermine Operations

TM 2320-20/12A Logistics Vehicle System

TM 92320-297-20 Logistics Vehicle System

TM 5-6115-545-12 Generator Set, Diesel

TC 10-2 Petroleum Terms, References, and Abbrevations

*U.S. Government Printing Office: 1997 - 432-836/60231

MARINE CORPS INSTITUTE

COURSE CONTENT ASSISTANCE REQUEST

MCI 13.34h MATH FOR MARINES

Use this form for questions you have about this course. Write out your question(s) and refer to the study unit, lesson, exercise item, or the review lesson exam item you are having a problem with. Before mailing, fold the form and staple it so that MCI's address is showing. Additional sheets may be attached to this side of the form. Your questions will be answered promptly by the Distance Instructor responsible for this course.

NAME	RANK	MOS

COMPLETE MILITARY ADDRESS (INCLUDE RUC IF KNOWN)	
	RUC

PHONE NUMBER	
DSN	COMMERCIAL (AREA CODE)

Your Question:

Instructor Response:

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MARINE CORPS INSTITUTE

COURSE EVALUATION QUESTIONNAIRE

MCI 13.34h MATH FOR MARINES

This questionnaire is extremely important to the Marine Corps Institute. The course that you have just completed has undergone extensive development and revision. As an integral part of the continued success of this course, YOUR HELP IS NEEDED. By completing this form, your responses may result in a need to revise the course.

Please take five minutes, complete the questionnaire and return it to MCI in the self-addressed envelope provided with your course materials. Additional comment sheets may be attached to this form. If you want to be contacted by the course instructor, please provide your name, rank, and phone number. Regardless of whether you want to be contacted or not, please enter your primary military occupational speciality (MOS).

NAME (Optional)	RANK	Enter Your MOS _____

Phone Number (Optional)	
DSN	COMMERCIAL (AREA CODE)

1. How long did it take you to complete this course including the review lesson examination? Check the box that applies.

   ☐ Less than three hours

   ☐ Three to six hours

   ☐ Seven to ten hours

   ☐ Eleven to fourteen hours

   If more than fifteen hours enter number of hours here: ________

2. Were the learning objectives stated at the beginning of each lesson clear? Check the box that applies.

   ☐ Yes ☐ No ☐ Not Sure

   If you checked "NO" please list below the study unit and lesson number for those unclear objectives.

   ____

   ____

3. Did the figures, that is illustrations, photographs, tables, charts, etc, clearly support the information/ text within the lesson? Check the box that applies.

☐ Yes ☐ No ☐ Not Sure

If you checked "NO," please list the figure numbers below.

______ ______ ______ ______ ______ ______ ______

4. Did the exercise at the end of a lesson or study unit really test your skills and knowledge gained by studying the lesson? Check the box that applies.

☐ Yes ☐ No ☐ Not Sure

If you checked "NO" please list the exercise item number the lesson number, and the study unit number below.

Question NO.	Lesson Number	Study Unit NO.

5. When you read the lesson the first time, did it make sense to you? Check the box that applies.

☐ Yes ☐ No If "No" please list the lesson(s) _____ _____ _____ _____ _____ _____

6. Would you recommend that a revision be made to any portion of this course? Check the box that applies.

☐ Yes ☐ No

If you checked YES, is the reason for your recommendation based on (Check all boxes that apply):

☐ Outdated procedures or process. Enter Study Unit Nos. ______

☐ Outdated equipment or material. Enter Study Unit Nos. ______

☐ Information not accurate. Enter Study Unit Nos. ______

☐ Other (Please describe)

7. Comments: Please attach separate sheet

STUDENT REQUEST/INQUIRY
MCI - R-11 (2/92)

DATE SENT: ____________________

COURSE NUMBER: 13.34h COURSE TITLE: MATH FOR MARINES

SECTION 1. STUDENT IDENTIFICATION

Instructions: Print or type rank, first name, middle initial, and last name, MOS, SSN, RUC, and military address clearly. Include ZIP CODE. Only Class III Reservists may use civilian address.

RANK	FIRST NAME MI LAST NAME	MOS	RUC	SOCIAL SECURITY NUMBER
MILITARY ADDRESS				

SECTION 2. STUDENT REQUEST/INQUIRY

Instructions: Check the appropriate box and fill in the appropriate spaces. For REGULAR AND CLASS II RESERVE MARINES THIS FORM MUST BE SIGNED BY THE COMMANDING OFFICER OR HIS REPRESENTATIVE e.g., TRAINING OFFICER.

☐ CHANGE	FROM / TO NAME ______ ______ RANK ______ ______ SSN ______ ______ RUC ______ ______	☐ MISSING DIPLOMA	THE COURSE WAS COMPLETED IN ____ 19____.
☐ MATERIALS	THE FOLLOWING MATERIALS ARE NEEDED LESSONS ______ MANUAL ______ ANSWER SHEETS ______ OTHERS ______	☐ EXTEND	(STUDENTS ARE ONLY ELIGIBLE FOR ONE EXTENSION PRIOR TO THEIR COURSE COMPLETION DATE (CDD)).
☐ EXAM OVERDUE	THE LAST LESSON WAS SENT IN ON ______	☐ REENROLL	(STUDENTS ARE ONLY ELIGIBLE FOR REENROLLMENT ONCE AND ONLY AFTER THEIR CCD. IF ALREADY REENROLLED AND DISENROLLED, A NEW ENROLLMENT MUST BE REQUESTED).
☐ MISSING RESULTS	THE EXAM WAS SENT IN ON ______ (IF NOT RECEIVED AT MCI A NEW EXAM WILL BE ISSUED).	☐ OTHER	OTHER (EXPLAIN):

NOTE: This form will not be returned by MCI. If the request is valid, transaction will show on next UAR or on MCI-R-1 Form.

SECTION 3. DATA REQUIRED BY THE PRIVACY ACT OF 1974 (5 U.S.C. 522A)

1. AUTHORITY: Title 5, USC, Sec. 301. Use of your Social Security Number is authorized by Executive Order 9397 of 22 Nov 43.

2. PRINCIPAL PURPOSE: The Student Request/Inquiry is used to transmit information concerning student participation in MCI courses.

3. ROUTINE USES: This information is used by MCI personnel to research student inquiries. In some cases information contained therein is used to update individual student records maintained by the Marine Corps Institute.

4. MANDATORY OR VOLUNTARY DISCLOSURE AND EFFECT ON INDIVIDUAL NOT PROVIDING INFORMATION: Disclosure is voluntary. Failure to provide information may result in the provision of incomplete service to your inquiry. Failure to provide your Social Security Number will delay the processing of your inquiry request.

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